

1. Let  $\mathbb{R}$  be set of real numbers. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ . Decide which of the following statements are true or false.

- (a) If  $f$  is increasing<sup>1</sup> then  $f$  is injective.
- (b) If  $f$  is increasing then  $f$  has an inverse.
- (c) If  $f$  is surjective then  $f$  is unbounded.
- (d) If  $f$  is unbounded then  $f$  is surjective.

2.  $f : (0, 1) \rightarrow \mathbb{R}$  be given by

$$f(x) = \begin{cases} \frac{2x-1}{2x} & \text{if } x \leq \frac{1}{2}. \\ \frac{2x-1}{2-2x} & \text{if } x \geq \frac{1}{2}. \end{cases}$$

Decide if  $f$  is a bijection.

3. Let  $\mathbb{N}$  be the set of natural numbers.

- (a) Show that  $n < 2^n$ .
- (b) There is an  $a \in \mathbb{N}$  and  $b \in \mathbb{N}$  such that  $n = 2^{a-1}(2b-1)$
- (c) Show that  $\text{Card}(\mathbb{N}) = \text{Card}(\mathbb{N} \times \mathbb{N})$

4. Decide if  $\text{Card}(A) < \text{Card}(B)$ ,  $\text{Card}(A) > \text{Card}(B)$ , or  $\text{Card}(A) = \text{Card}(B)$  when

- (a)  $A = (0, 1)$  and  $B = [0, 1]$
- (b)  $A = \mathbb{N}$  and  $B = \mathbb{R}$ .

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<sup>1</sup>i.e.  $f(x) < f(y)$  whenever  $x, y \in \mathbb{R}$  and  $x < y$