Pigeon Hole and Inclusion-Exclusion

1. If a and p are relatively prime, then there exists some $x \in \{1, \ldots, p-1\}$ such that $ax \equiv 1 \mod p$.

2. Given five integer points[1] in the plane, the midpoint of the segment joining some pair of them is also an integer point.

3. Siva collects homework papers from 17 students who come to his class and returns them at random for friendly grading by the students. What is the Probability that no student receives his or her paper ?

4. We roll a six-sided die until each of the numbers one through five have appeared at least once. What is the probability that we succeed in the first n rolls?

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[1] an integer point in a plane is (x, y) where $x, y \in \mathbb{Z}$.