1. Let $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ be two sequence of real numbers. Decide (with an appropriate choice of proof) if

$$\limsup_{n\to\infty}(a_n + b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n.$$

and

$$\liminf_{n\to\infty}(a_n + b_n) \geq \liminf_{n\to\infty} a_n + \liminf_{n\to\infty} b_n.$$

2. Let $\{a_n\}_{n\geq 1}$ be a sequence of positive numbers.
(a) Show that

$$\liminf_{n\to\infty} \frac{a_{n+1}}{a_n} \leq \liminf_{n\to\infty} \sqrt[n]{a_n} \leq \limsup_{n\to\infty} \sqrt[n]{a_n} \leq \limsup_{n\to\infty} \frac{a_{n+1}}{a_n}$$

(b) Can you construct an example of a sequence where all the inequalities above are strict and all quantities are real numbers ?.

1. Let $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ be two sequence of real numbers. Let $L = \liminf_{n\to\infty} a_n$ and $M = \liminf_{n\to\infty} b_n$. Decide (with an appropriate choice of proof) if

$$\liminf_{n\to\infty}(a_n + b_n) \geq L + M.$$

2. Let $\{a_n\}_{n\geq 1}$ be a sequence of positive numbers.
(a) Show that

$$\limsup_{n\to\infty} \sqrt[n]{a_n} \leq \limsup_{n\to\infty} \frac{a_{n+1}}{a_n}$$

(b) Can you construct an example of a sequence where all the inequalities above are strict and all quantities are real numbers ?.