Writing of Mathematics **Quiz 6** **Semester I 2018/19**

Your name: October 29th, 2018

1. Let $f : \mathbb{R} \to \mathbb{R}$.

   (a) Define using Logical notation what is meant by saying "*f is a bounded function.*

   (b) Negate the above statement.

   (c) Let $h, g : \mathbb{R} \to \mathbb{R}$. Decide whether the following statement is true or false[1]:

   If $h + g$ is a bounded function then h and g are also bounded functions.

Solution:(a) $f : \mathbb{R} \to \mathbb{R}$ is said to be bounded if $\exists M > 0$ such $\mid f(x) \mid \leq M$ for all $x \in \mathbb{R}$.

Solution:(b) for all $M > 0$, there exists an $x \in \mathbb{R}$ such that $\mid f(x) \mid > M$.

Solution:(c) The statement is false. let $h : \mathbb{R} \to \mathbb{R}$ be given by $h(x) = x$ and $g : \mathbb{R} \to \mathbb{R}$ be given by $g(x) = -x$. Note that for any $M > 0$, for $x = M + 1$ $\mid h(x) \mid = M + 1 > M$ and $\mid g(x) \mid = M + 1 > M$. So h, g are both not bounded functions. But $h + g(x) = 0$ for all $x \in \mathbb{R}$ and is trivally a bounded function.

2. Let $f : \mathbb{R} \to \mathbb{R}$. We say $\lim_{x \to 0} f(x) = 0$ if

$$\textit{For every } \epsilon > 0 \textit{ there exists } \delta > 0 \textit{ such that } \mid f(x) \mid < \epsilon \textit{ whenever } \mid x \mid < \delta. \tag{1}$$

Consider the following statements:

(a) For every $\epsilon > 0$ there exists $\delta > 0$ such that for all $x \in \mathbb{R}$, $\mid x \mid < \delta$ implies $\mid f(x) \mid < \epsilon$.

(b) For every $\delta > 0$ there exists $\epsilon > 0$ such that for all $x \in \mathbb{R}$, $\mid x \mid < \delta$ implies $\mid f(x) \mid < \epsilon$.

(c) There exists $\delta > 0$ such that for all $\epsilon > 0$ and for all $x \in \mathbb{R}$, $\mid x \mid < \delta$ implies $\mid f(x) \mid < \epsilon$.

(d) For every $\epsilon > 0$ and for all $x \in \mathbb{R}$, there exists $\delta > 0$ such that $\mid x \mid < \delta$ implies $\mid f(x) \mid < \epsilon$.

For each of the above versions (a)-(d) that are NOT equivalent to (1) provide an example of f that satisfy them but not (1).

Solution: See Class board Photos.

---

[1]If true then please provide a proof. If false then please provide a counter-example.