Writing of Mathematics	$\mathbf{Quiz} \ 4$	Semester I 2018/19
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Your name: Solution

1. Consider the groups \mathbb{Z}_9^{\times} of invertible residues modulo 9 under multiplication and \mathbb{Z}_{12}^{\times} of invertible residues modulo 12 under multiplication.

- (a) List the elements of \mathbb{Z}_9^{\times} and \mathbb{Z}_{12}^{\times}
- (b) Decide if either group is cyclic.

Solution:

(a) Elements of, \mathbb{Z}_n^{\times} are those elements in $\{k: 0 \leq k \leq n-1\}$ that are coprime to n. So

$$\mathbb{Z}_9^{\times} = \{1, 2, 4, 5, 7, 8\}$$

and

$$\mathbb{Z}_{12}^{\times} = \{1, 5, 7, 11\}.$$

(b) The cyclic subgroups generated by each element in \mathbb{Z}_9^\times are:

The group \mathbb{Z}_9^{\times} is cyclic as 2 and 5 are generators.

The cyclic subgroups generated by each element in \mathbb{Z}_{12}^{\times} are:

$$\begin{split} &\langle 1\rangle = \{1\} \\ &\langle 5\rangle = \{1,5\} \\ &\langle 7\rangle = \{1,7\} \\ &\langle 11\rangle = \{1,11\} \end{split}$$

5,7,11 all have order 2 hence the group is not cyclic.