Writing of Mathematics **Quiz 2** **Semester I 2018/19**

Your name: **Solution** OCtber 4th, 2018

1. Suppose $\{z_n\}_{n\geq 1}$ is a sequence of real numbers. Without using the ratio or root test decide whether the series $\sum_{n=1}^{\infty} z_n$ converges when

$$z_n = \frac{\sqrt{n}}{2n^3 - 1}$$

Solution: For $n \in \mathbb{N}$ observe that

$$0 \leq z_n = \frac{\sqrt{n}}{2n^3 - 1} = \frac{\sqrt{n}}{n^3}\frac{1}{2 - \frac{1}{n^3}} = \frac{1}{n^{2.5}}\frac{1}{2 - \frac{1}{n^3}}$$

For all $n \geq 1$, note that $2 - \frac{1}{n^3} \geq 2 - \frac{1}{1^3} = 1$. So,

$$\frac{1}{2 - \frac{1}{n^3}} \leq 1.$$

Therefore $0 \leq z_n \leq \frac{1}{n^{2.5}}$, for all $n \in \mathbb{N}$. This implies that

$$\mid z_n \mid \leq \frac{1}{n^{2.5}}, \text{ for all } n \in \mathbb{N}.$$

As $\sum\limits_{n=1}^{\infty} \frac{1}{n^{2.5}}$ converges[1], by the comparison test[2], we have that

$$\sum_{n=1}^{\infty} z_n \text{ also converges.}$$

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[1] $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$ converges for all $p > 1$.

[2] Let $\{z_n\}_{n\geq 1}$ and $\{y_n\}_{n\geq 1}$ be sequences of two real numbers. Let $N \geq 1$. Suppose $\mid z_n \mid \leq y_n$ for all $n \geq N$ and $\sum\limits_{n=1}^{\infty} y_n$ converges then $\sum\limits_{n=1}^{\infty} z_n$ converges.

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1. Suppose $\{z_n\}_{n\geq 1}$ is a sequence of real numbers. Without using the ratio or root test decide whether the series $\sum_{n=1}^{\infty} z_n$ converges when

$$z_n = \frac{n^2 - n + 1}{n^3 + 1}$$

Solution: For $n \in \mathbb{N}$,

$$z_n = \frac{n^2 - n + 1}{n^3 + 1} = \frac{1}{n}\frac{n^3 - n^2 + n}{n^3 + 1} \geq \frac{1}{n}\frac{n^3 - n^2 + 1}{n^3 + 1} = \frac{1}{n}(1 - \frac{n^2}{n^3 + 1}) = \frac{1}{n}(1 - \frac{1}{n + \frac{1}{n^2}})$$

Now for all $n \geq 2$,

$$n + \frac{1}{n^2} \geq 2.$$

So for all $n \geq 2$,

$$1 - \frac{1}{n + \frac{1}{n^2}} \geq 1 - \frac{1}{2} = \frac{1}{2}.$$

Therefore,

$$z_n \geq \frac{1}{n} \cdot \frac{1}{2} \geq 0$$

As $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ diverges[3]. By the comparison test[4], $\sum\limits_{n=1}^{\infty} \frac{1}{2n}$ also diverges, and hence again by the comparison test

$$\sum_{n=1}^{\infty} z_n \text{ diverges to } \infty.$$

[3] $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$ diverges to ∞ for all $p \leq 1$.

[4] Let $\{z_n\}_{n\geq 1}$ and $\{y_n\}_{n\geq 1}$ be sequences of two real numbers. Let $N \geq 1$. Suppose $0 \leq y_n \leq z_n$ for all $n \geq N$ and $\sum\limits_{n=1}^{\infty} y_n$ diverges to ∞ then $\sum\limits_{n=1}^{\infty} z_n$ diverges to ∞.