

Homework 8*Due Date:* October 17th, 2018*Problems Due:* 1

1. Let $n \geq i$, and $\phi : \mathbb{Z} \rightarrow \mathbb{Z}_n$ be given by $\phi(i) = i \pmod n$. Show that

$$\phi(i + j) = \phi(i) +_n \phi(j) \text{ and } \phi(i \cdot j) = \phi(i) \cdot_n \phi(j).$$

2. Write out the cycle decomposition of each element in S_4 . Find the order of each element.
3. Find all elements of $G = \mathbb{Z}_9$ which generates G as a cyclic group. Find all subgroups of G and also find a generator in each case.
4. Let p be a prime number. Let $a \in \mathbb{N}$ not divisible by p . Show that $f : \mathbb{Z}_p \rightarrow \mathbb{Z}_p$ given by $f(x) = ax$ is a bijection.
5. Define f and g from \mathbb{Z}_n to \mathbb{Z}_n by $f(\bar{x}) = \bar{x} +_n \bar{a}$ and $g(\bar{x}) = \bar{x} \cdot \bar{a}$
 - (a) Give a complete description of the functional digraph of f .
 - (b) Draw the functional digraph of g for the case $(n, a) = (19, 4)$ and the case $(n, a) = (20, 4)$. Describe a property of the digraph that is true whenever n is prime and false whenever n is not prime.

Extra Credit Puzzles:

1. *The coins Problem.* Given several piles of 1 Rupee coins from the ISI-Canteen cashier, we create a new collection by removing one coin from each old pile to make a new pile. Each original pile shrinks by one. That is, for example: if the original collection had four piles of 1,1,2,5 then the new set of piles will have 1,4,4.
 - (a) Which lists of sizes (order is not important) are unchanged under this operation ?
 - (b) Let S_n be the set of non decreasing lists summing to n . Let $f : S_n \rightarrow S_n$ be the function defined on S_n by the operation above.
 - (i) Draw the functional digraph of f when $n = 6$.
 - (ii) Determine all values of n such that f is injective. Determine all values of n such that f is surjective.
2. *The darts Problem.* Suppose a dart board has regions with values a and b , where a and b are natural numbers that are relatively prime. What is the largest integer k that cannot be achieved by summing the values of thrown darts? We seek k such that $ma + nb = k$ has no solutions in nonnegative integers m, n , but $ma + nb = j$ does have such a solution whenever j is an integer larger than k .

¹**Office hours:** I will be in my office from 8:15-9am on Tue, Wed, Thu and from 2-3pm on Wed to answer any questions that you may have. Please feel free to drop by during these times to clarify any doubts that you may have.

6. In the following table values of x and the corresponding y values are given:

x	y
0	2
1	3
2	4
3	6
5	7

- (a) Find the least square regression line $y = ax + b$
- (b) Estimate the value of y when $x = 4, 6$.
7. The mayor of the city of Bellary, Daxita, wants to buy more AC buses for the city. She commissions the transportation department to study the relationship between the temperature and the number of passengers that ride the main AC bus line in order to better serve their customers. The manager, Rahi, recorded the temperature at the beginning of the hour, and then had the bus driver, Sakshi, record the number of passengers that boarded the bus throughout the hour. Their findings are listed below.

Temperature (x)	Passengers(y)
42	173
37	149
46	185
30	123
50	201
43	174
43	175
46	188
46	186
49	198

- (a) Fill in the following table

Variable	Mean	SD
Temperature		
Passengers		

- (b) Use the above information to obtain the least square equation $y = ax + b$.
- (c) Interpret the slope.
- (d) What is the prediction error for the entry temperature = 42°C, passengers = 173?
- (e) Can you predict the number of passengers when temperature is at 55°C?
- (f) Is there a relation between the prediction errors and the temperature?

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