WM18¹ Writing of Mathematics http://www.isibang.ac.in/~athreya/Teaching/wom18

> Homework 7 Due Date: October 11th, 2018 Problems Due: 1

- 1. Let \mathbb{Z} be the set of integers. Consider 2-dimensional lattice given by $\mathbb{Z} \times \mathbb{Z}$. Let 0 < c and 0 < d. A lattice path from (0,0) to (c,d) in $\mathbb{Z} \times \mathbb{Z}$ is a path that starts at (0,0) and that moves up or right to reach (c,d). Find the number of such paths from (0,0) to (c,d).
- 2. Let $k, n \in \mathbb{N}$ such that k < n. Find the number of non-negative integer solutions to $\sum_{i=1}^{k} x_i = n$
- 3. Let G be a set endowed with an operation \cdot . It is given that there is an $e \in G$ such that $g \cdot e = g$ for all $g \in G$ and for all $g \in G$ there is a $h \in G$ such that $h \cdot g = e$. Is (G, \cdot) necessarily a group ?
- 4. Suppose two teams play a series of games, each producing a winner and a loser, until one team has won two more games than the other. Let G be the total number of games played. Assuming each team has chance 0.5 to win each game, independent of results of the previous games. Find the expected value of G.
- 5. Let $n \ge 2$ and $\sigma : \{1, 2, \dots, n\} \to \{1, 2, \dots, n\}$ be a bijection. We say that a subset T of \mathbb{R}^n is symmetric if

 $(x_1, x_2, \dots, x_n) \in T \iff (x_{\sigma(1)}, x_{\sigma(2)}, \dots, x_{\sigma(n)}) \in T$

for all $(x_1, x_2, \ldots, x_n) \in T$. For any symmetric set T, a function $f: T \to \mathbb{R}$ is symmetric if

$$f(x_1, x_2, \ldots, x_n) = f(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)})$$

for all $(x_1, x_2, \ldots, x_n) \in T$.

- (a) Let n = 3. Provide three examples of symmetric sets in \mathbb{R}^3 which are proper subsets of \mathbb{R}^3 .
- (b) Let n = 3. Provide three examples of symmetric functions in \mathbb{R}^3 which are different from the identity.

Extra Credit Puzzles:

- 1. B.Math (hons.) first year class room has 40 seats numbered 1 through 40 for its 40 students. Students have all been assigned seats by the office. Siva asks students to enter the class in chronological order. Then just as the first student enters, Siva chooses a random number between 1 and 40 and asks the student to sit there.Each subsequent student takes his or her seat if available, otherwise takes one of the remaining seats at random.
 - (a) What is the probability that the last student will sit in his/her own seat number 40?
 - (b) What is the probability that the last bt one student will sit in his/her own seat number 39?

¹Office hours: I will be in my office from 8:15-9am on Tue,Wed, Thu and from 2-3pm on Wed to answer any questions that you may have. Please feel free to drop by during these times to clarify any doubts that you may have.