Homework 3

Due Date: August 29th, 2018

1. For the following statements below write down their negations in words (like the statements).
   (a) If A holds, then B holds.
   (b) A is true only if B is true.
   (c) A is true whenever B is true.
   (d) A is false only if B is false.
   (e) A is a necessary condition for B.
   (f) A is necessary and sufficient for B.
   (g) A holds if and only if B holds

2. Negate the below statements and express the negations in English avoiding the use of negation words whenever possible.
   (a) All classrooms in the main building have at least one chair that is broken.
   (b) No classroom in the ground floor has only chairs that are not broken.
   (c) Every student in this class has taken Mathematics or Physics in Class XII.
   (d) Every student in this class has taken Mathematics and Biology in Class XII.
   (e) In every batch of B.Math (hons) there is a student who has taken neither Mathematics nor Biology in high school.

   Example 1: We introduce the following **Logical Notation:** $\forall$ to mean for all; $\exists$ to mean there exists; $\implies$ to mean implies; and $\iff$ to mean equivalent. Here is an example of usage of notation.

   Statement : $f(x, y) \neq 0$ whenever $x \neq 0$ and $y \neq 0$.
   Statement in logical Notation: $\forall x, y \in \mathbb{R}$, $[x \neq 0, y \neq 0 \implies f(x, y) \neq 0]$

3. As indicated in first example: (i)Translate the following sentences into logical notation, (ii) negate the statement using logical rules, and (iii) translate the negated statement back into English.
   (a) For all $M \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that $| f(x) | \geq M$.
   (b) For all $M \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that for all $y > x$ we have $f(y) > M$.
   (c) For all $x \in \mathbb{R}$ there exists $y \in \mathbb{R}$ such that $f(y) > f(x)$.
   (d) For every $\epsilon > 0$ there exists $x_0 \in \mathbb{R}$ such that $| f(x) | < \epsilon$ for all $x > x_0$.
   (e) For every $\epsilon > 0$ there exists $\delta > 0$ such that $| f(x) - f(y) | < \epsilon$ whenever $| x - x_0 | < \delta$.