Homework 2
Due Date: August 8th, 2018

1. Let x, y, u, v be real numbers. Prove that
$$(xu + yv)^2 \leq (x^2 + y^2)(u^2 + v^2)$$
and determine precisely when equality will hold in the above statement.

2. Let $S = \{x \in \mathbb{R} : x(x-1)(x-2)(x-3) < 0\}$. Let $T = (0,1)$ and $U = (2,3)$. Obtain a simple set equality relating S, T, U.

3. Let $a \in \mathbb{R}$. We shall use the convention $(-\infty, a) = \{x \in \mathbb{R} : x < a\}$. Let $n \in \mathbb{N}$ and $a_i \in \mathbb{R}$ for $1 \leq i \leq n$ with $a_i < a_{i+1}$ for $1 \leq i \leq n-1$. Express $S = \{x \in \mathbb{R} : \prod_{i=1}^{n}(x - a_i) < 0\}$ using the notation for intervals.

4. Let $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 100\}$ and $T = \{(x,y) \in \mathbb{R}^2 : x + y \leq 14\}$. Graph the region $S \cap T$ and count the number of elements in $S \cap T \cap \mathbb{Z}^2$.

5. Let $D \subset \mathbb{R}$ be the domain of $f : D \to \mathbb{R}$. In each of the following cases find the domain D.
   (a) Let $h(x) = \dfrac{\sqrt{3-x}}{2+x}$ and $f(x) = \sqrt{h(x)}$
   (b) $f(x) = 7 + \sqrt{25 - \frac{(x+1)^2}{4}}$.

6. Let $f, g : \mathbb{R} \to \mathbb{R}$. Determine which of the following statements are true. If true then provide a proof and if false then provide a counter example.
   (a) If f, g are bounded then $f + g$ is bounded[1]
   (b) If $f + g$ is bounded then f and g are bounded.
   (c) If both $f + g$ and fg are bounded then f and g are bounded.

Other Problems not due:

1. A positive integer is palindromic if reversing the digits of its base 10 representation does not change the number. Why is every palindromic integer with an even number of digits divisible by 11 ?

2. Let $x \in [0,1]$ and we push x^2 in the calculator repeatedly. Let the sequence of numbers generated be denoted by x_n. Can you identify where the sequence *tends* to as n gets *large* ? What happens if we replace x^2 by a general quadratic function ?

[1] $T : \mathbb{R} \to \mathbb{R}$ is said to be bounded if there is an $M > 0$ such that $| T(x) | \leq M$ for all $x \in \mathbb{R}$