$$y_n = \sum_{k=0}^{n} \frac{1}{k!} \qquad x_n = \left(1+\frac{1}{n}\right)^n$$

1 $y_n \to e$ as $n \to \infty$

2 $x_n \le y_n \quad \forall\, n \ge 1$

$$\boxed{\bar{x} = \limsup_{n\to\infty} x_n \le \limsup_{n\to\infty} y_n = e} \quad - \; 1'$$

$$x_n = \left(1+\frac{1}{n}\right)^n = \sum_{k=0}^{n} {}^{n}C_k \left(\frac{1}{n}\right)^k = \sum_{k=0}^{n} \frac{1}{k!} \frac{\prod_{i=0}^{k-1}(n-i)}{n^k}$$

$$x_n = \sum_{k=0}^{n} \frac{1}{k!} \prod_{i=0}^{k-1} \left(1 - \frac{i}{n}\right)$$

let $m \geq 1$

$n \geq m+1$ $\quad x_n = \sum_{k=0}^{m} \frac{1}{k!} \left(\prod_{i=0}^{k-1} \left(1 - \frac{i}{n}\right)\right) + \text{"} \sum_{k=m+1}^{n} \frac{1}{k!} \cdots \text{"}$

$$x_n \geq \sum_{k=0}^{m} \frac{1}{k!} \prod_{i=0}^{k-1} \left(1 - \frac{i}{n}\right) + 0 \quad \text{--- } (1)$$

$\frac{1}{n} \to 0$ as $n \to \infty \Rightarrow \frac{i}{n} \to 0$ as $n \to \infty$ $\left\{ \begin{array}{l} \text{Algebra} \\ \text{of} \\ \text{limits} \end{array} \right\}$ $\Rightarrow$

$1 \leq i \leq m$

$$\alpha_n := \sum_{k=0}^{m} \frac{1}{k!} \prod_{i=0}^{k-1} \left(1 - \frac{i}{n}\right) \xrightarrow{\text{as } n \to \infty} \sum_{k=0}^{m} \frac{1}{k!} \equiv y_m$$

From 1 $\quad x_n \geq \alpha_n \quad \forall n \geq m$

$$\boxed{\underline{x} := \liminf_{n \to \infty} x_n \geq \liminf_{n \to \infty} \alpha_n = y_m} \quad \text{— } 2$$

From 0 and 2 $\quad y_m \leq \underline{x} \leq \bar{x} \leq e \quad \forall m \geq 1$

$m \to \infty$ [illegible] inequality

as $y_m \to e$ and rest are constant sequences

$$e \leq \underline{x} \leq \bar{x} \leq e$$

$\Rightarrow x_n \to e$ as $n \to \infty$