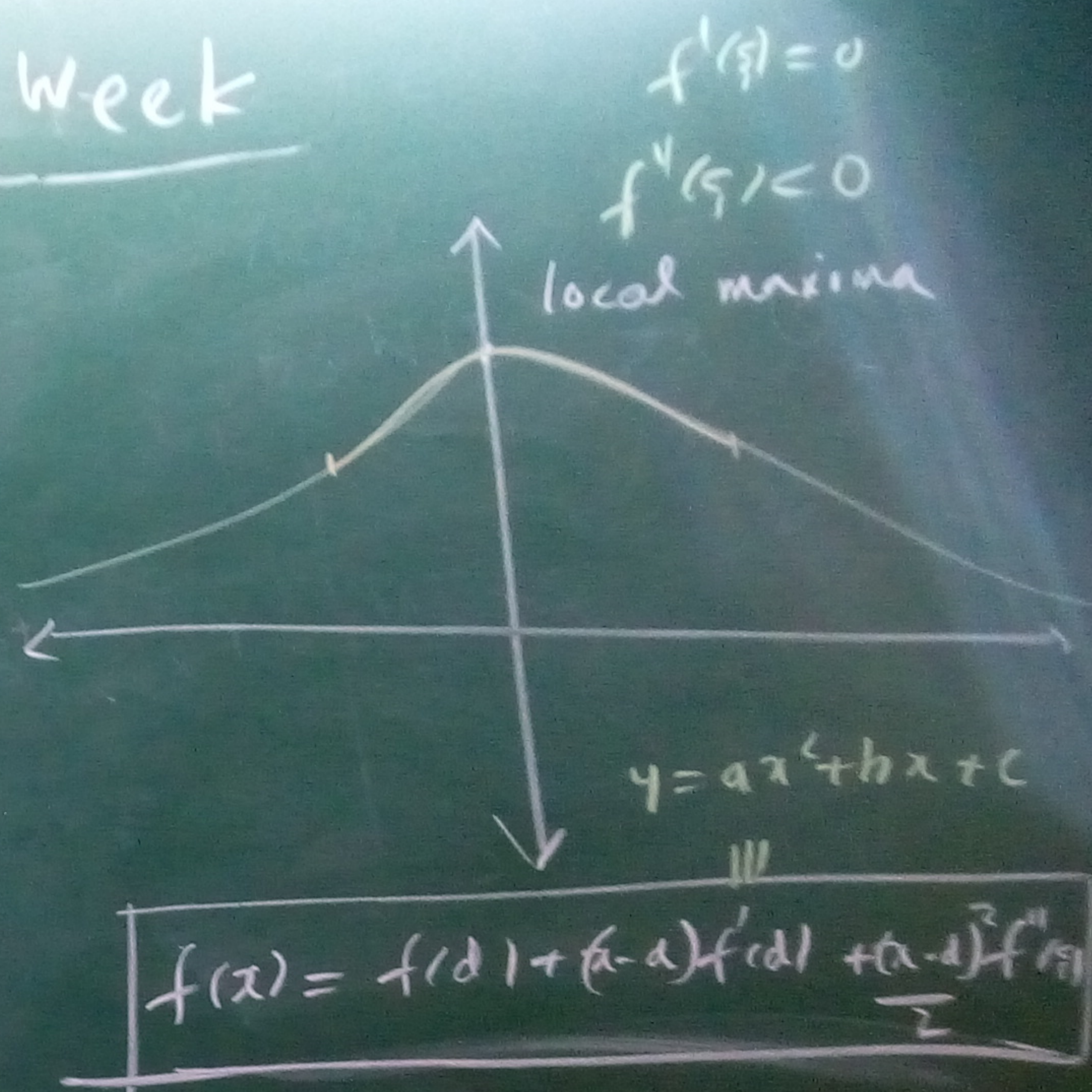


2nd Week / 3rd Week

$$f(x) = e^{-x^2/2}$$

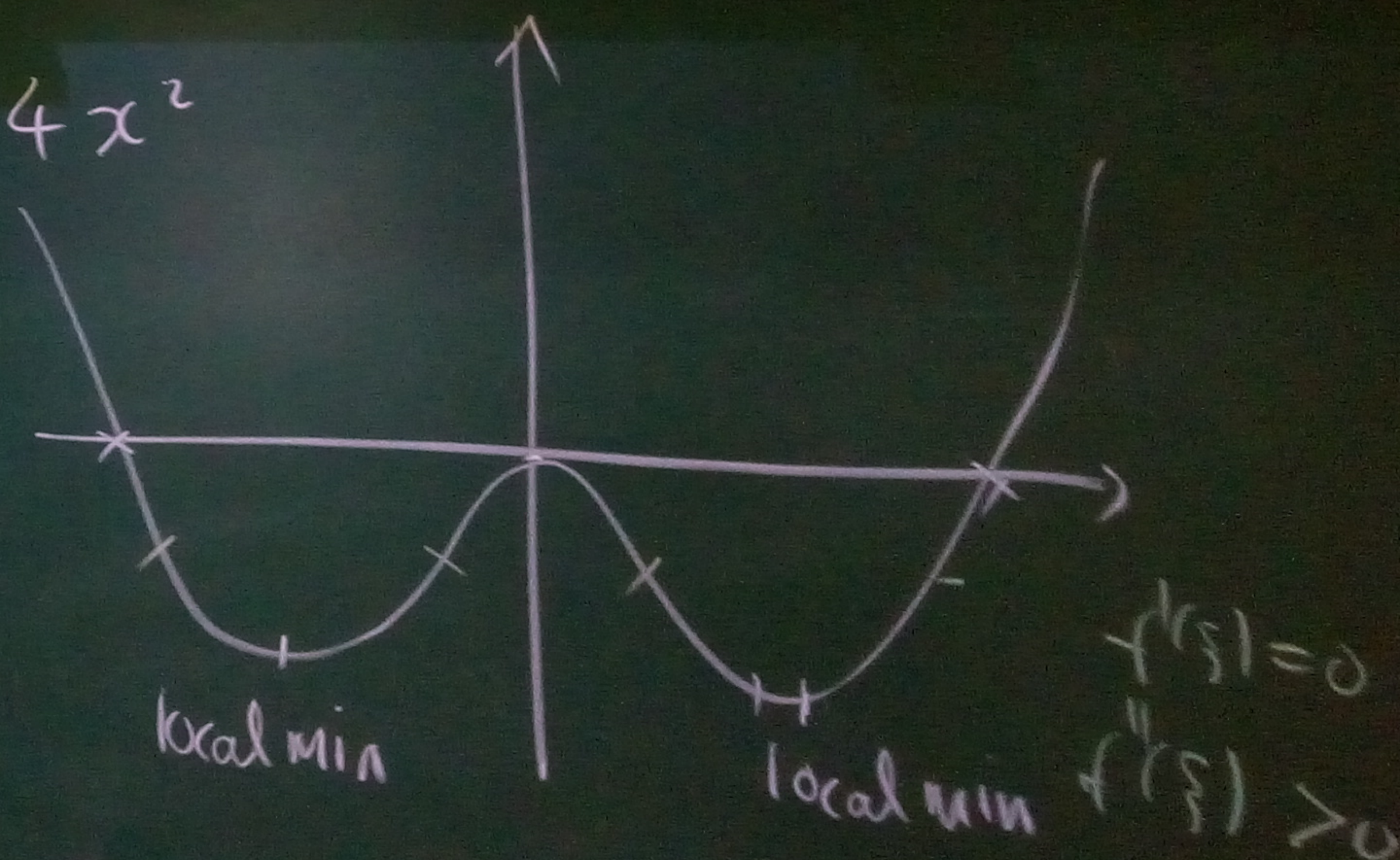
$$f: \mathbb{R} \rightarrow \mathbb{R}$$

- zero of f
- $f'(\cdot)$ - Behavior at $\infty, -\infty$
- $f''(\cdot)$



$$f(x) = x^4 - 4x^2$$

- f - zero
- $f'(\cdot)$
- $f''(\cdot)$



$$f: \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$1 \text{ (a) } h(x, y) = x + y$$

(i)

$$x + y = 9$$

$$x + y = 100$$

(ii)

$$x + y = 0$$

$(1, -1, 0)$
 $(0, 0, 0)$
 $(1, 2, 3)$

$$h(x, y) = x + y$$

$$\rightarrow x + y = 0$$

1 (b)

$$L = 9 \quad L = 100 \quad \neq$$

$$L = -9 \quad (i)$$

$$\{(x, y) \mid -y^2 = -9\}$$

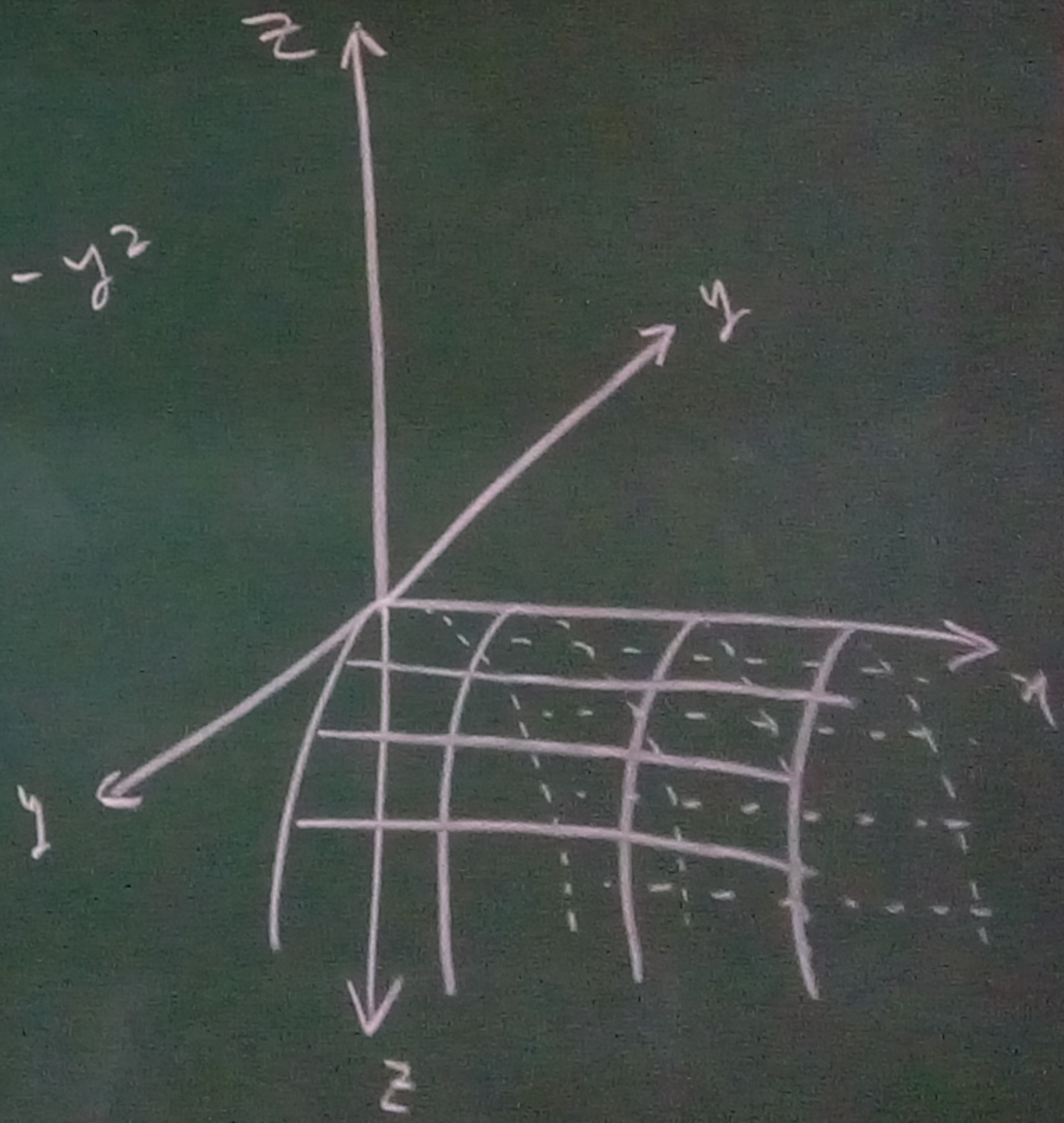
$$\{(x, y) \mid y^2 = 9\} \quad (ii)$$

$$z = -y^2$$

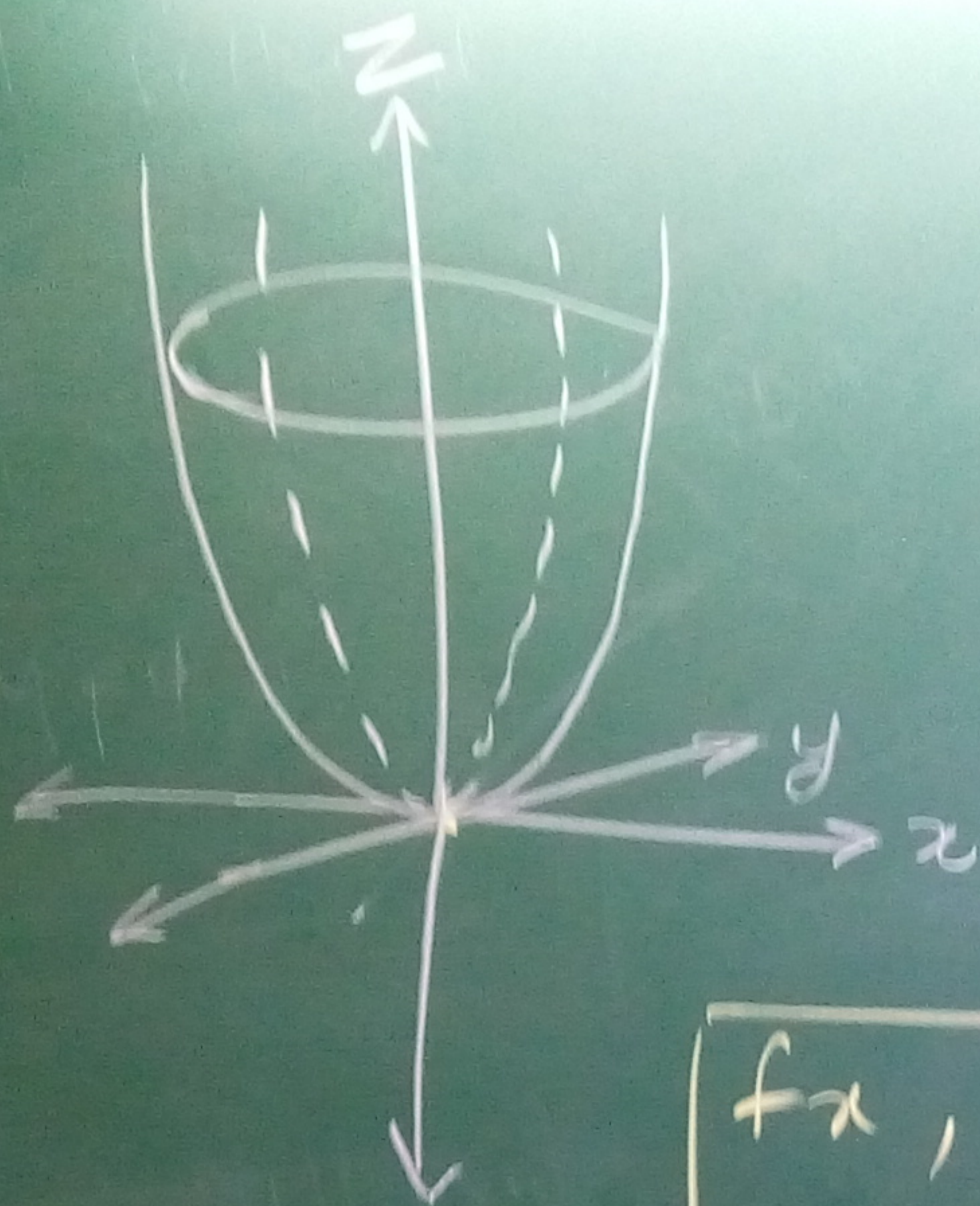
$$L = 9$$

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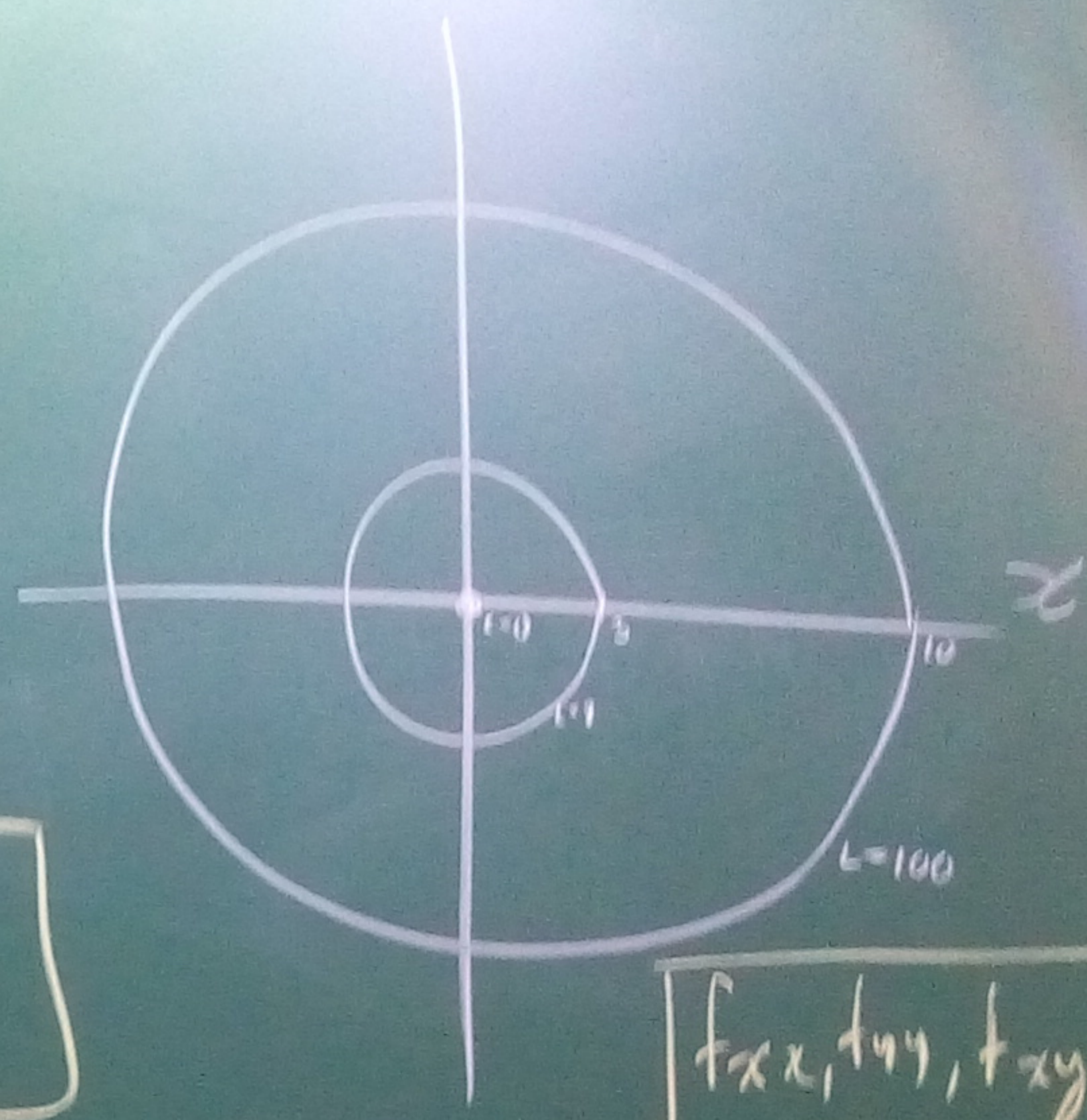
$$L = -9$$



1c



f_x, f_y



f_{xx}, f_{yy}, f_{xy}

3

$$h_x(a, b) = 1$$

$$h_y(a, b) = 1 \quad \forall a, b \in \mathbb{R}$$

$$f_x(a, b) = 0$$

$$f_y(a, b) = -2b$$

$$g_x(a, b) = 2a$$

$$g_y(a, b) = 2b$$

$$q_x(a, b) = 2a$$

$$q_y(a, b) = -2b$$

$$6 \quad f(x, y) = x^2 - 2xy + 6y^2 + 4x - 4y$$

(a)

$$f_x(a, b) = 2a - 2b + 4$$

$$f_y(a, b) = -2a + 12b - 4$$

(b) Critical points

$$① - 2a - 2b + 4 = 0$$

$$② - 2a + 12b - 4 = 0$$

① + ② $\Rightarrow b = 0, a = -2$ is only critical point.

(c) $(-2, 0)$ is it a local min?
max?
saddle point?

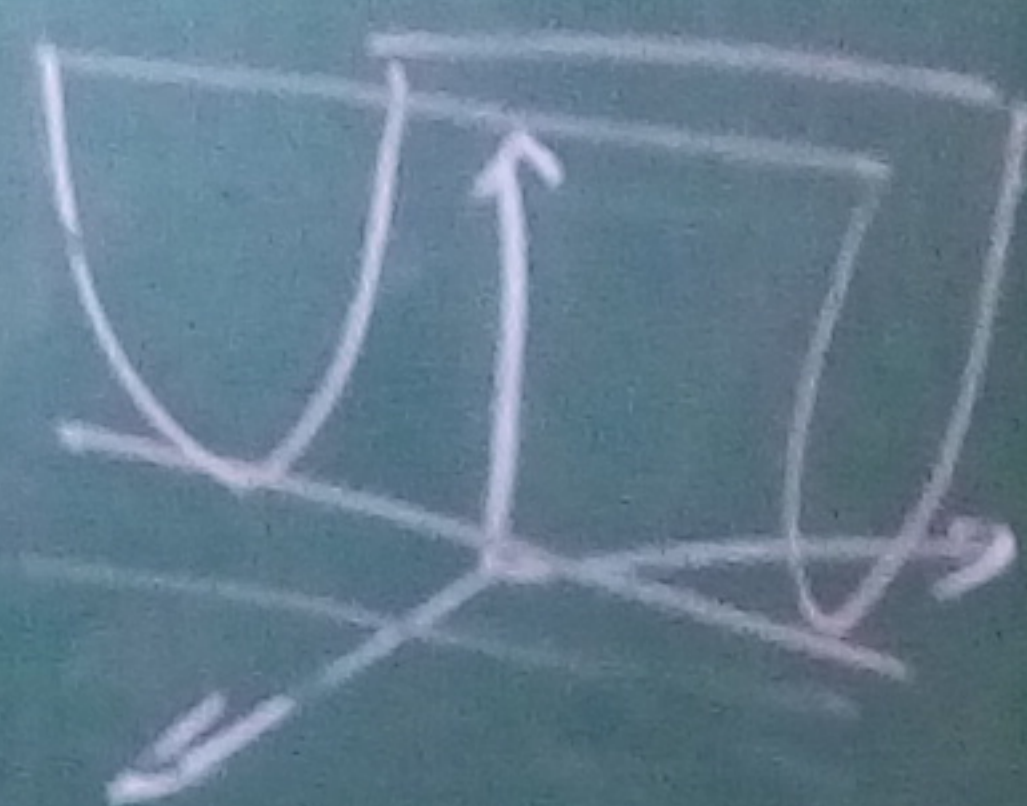
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⑦ contd.

$$① b^2 - 4ac = 0$$

$$f(x,y) = a \left(x + \frac{by}{2a} \right)^2$$



② $a > 0$, minima happen at

$$(x,y) : x + \frac{by}{2a} = 0$$

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$f(x,y)$

Note

7

$$\begin{aligned}
 f(x,y) &= (x-y)^2 + 5y^2 + 4x - 4y \\
 &= (x-y)^2 + 4(x-y) + 5y^2 \\
 &= (x-y+2)^2 - 4 + 5y^2
 \end{aligned}$$

Note $f(x,y) \geq 0 - 4 + 0 = -4$

$$f(-2,0) = (-2-0+2)^2 - 4 + 5(0)^2 = -4$$

$(-2,0)$ is a global minimum $\square$

$$\begin{aligned}
 \textcircled{7} \quad f(x,y) &= ax^2 + bxy + cy^2 \\
 (a \neq 0) \quad &= a \left[x^2 + \frac{b}{a}xy \right] + cy^2 \\
 &= a \left[\left(x + \frac{b}{2a}y \right)^2 - \frac{b^2}{4a^2}y^2 \right] + cy^2 \\
 &= a \left(x + \frac{b}{2a}y \right)^2 - \frac{1}{4a} (b^2 - 4ac) y^2
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \quad h(x,y) &= 1 - x^2 - y^2 \\
 \therefore h(x,y) &\leq 1
 \end{aligned}$$

$\textcircled{7}$ Cont. A

$$\textcircled{1} \quad b^2 - 4ac < 0$$

$$f(x,y) =$$

$$\textcircled{x} \quad a > 0, \quad \text{min}$$

$$(x,y) = x + y$$

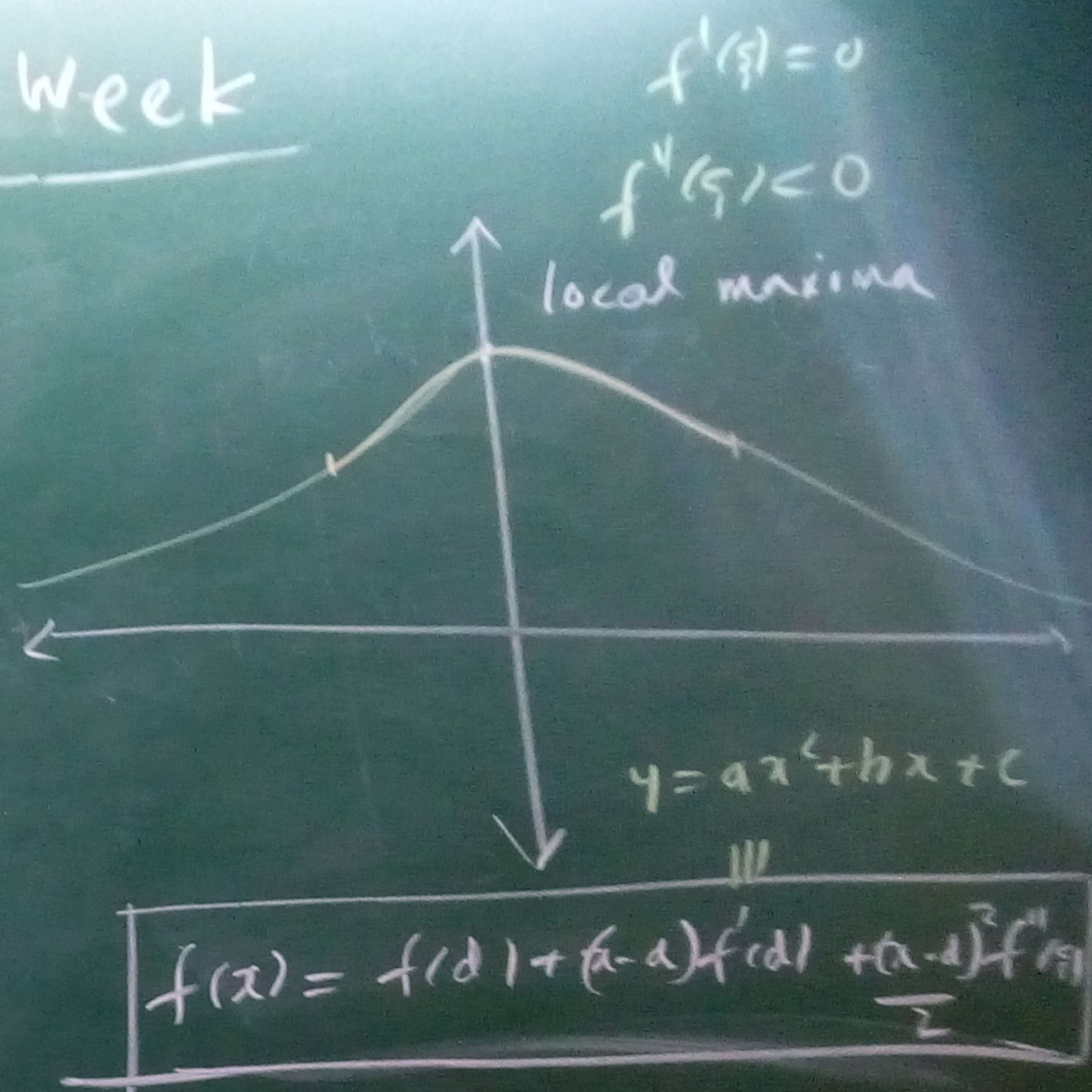
$$\textcircled{+} \quad a < 0$$

2nd Week / 3rd Week

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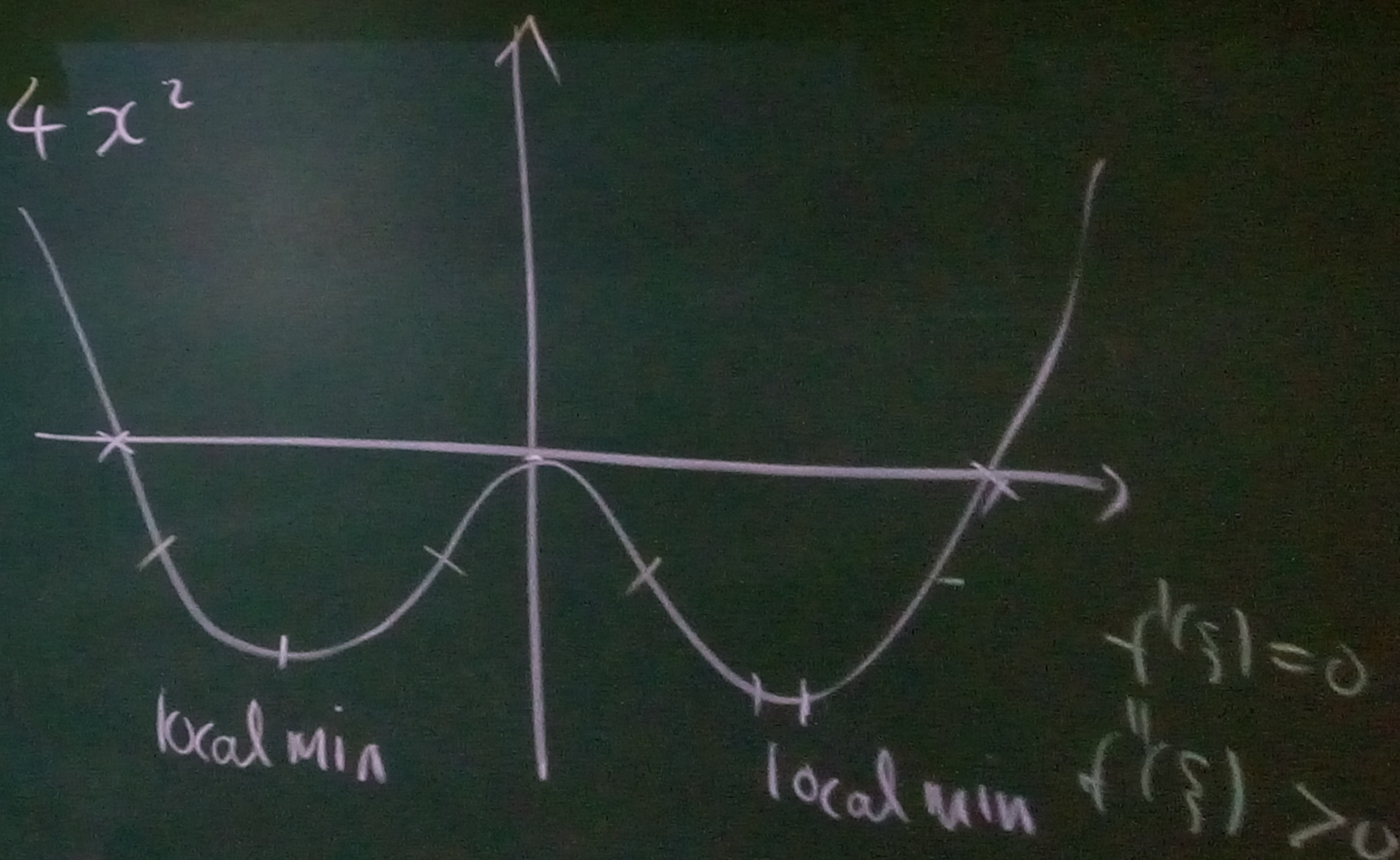
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$$\begin{aligned} (1, -1, 0) \\ (0, 0, 0) \\ (1, 2, 3) \end{aligned}$$

$$h(x, y) = x + y$$

$$h(x, y) = 0 \Rightarrow x + y = 0$$

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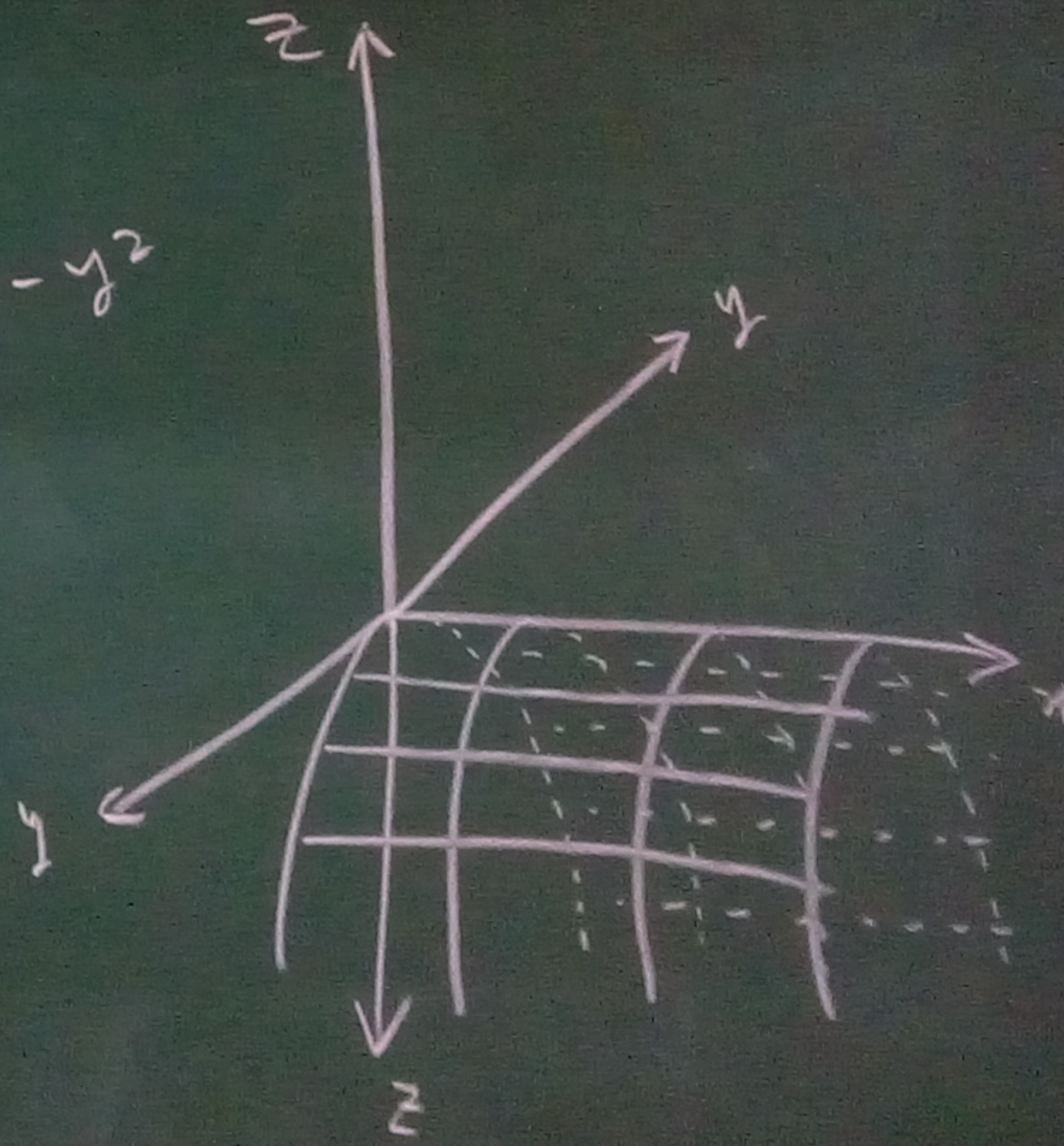
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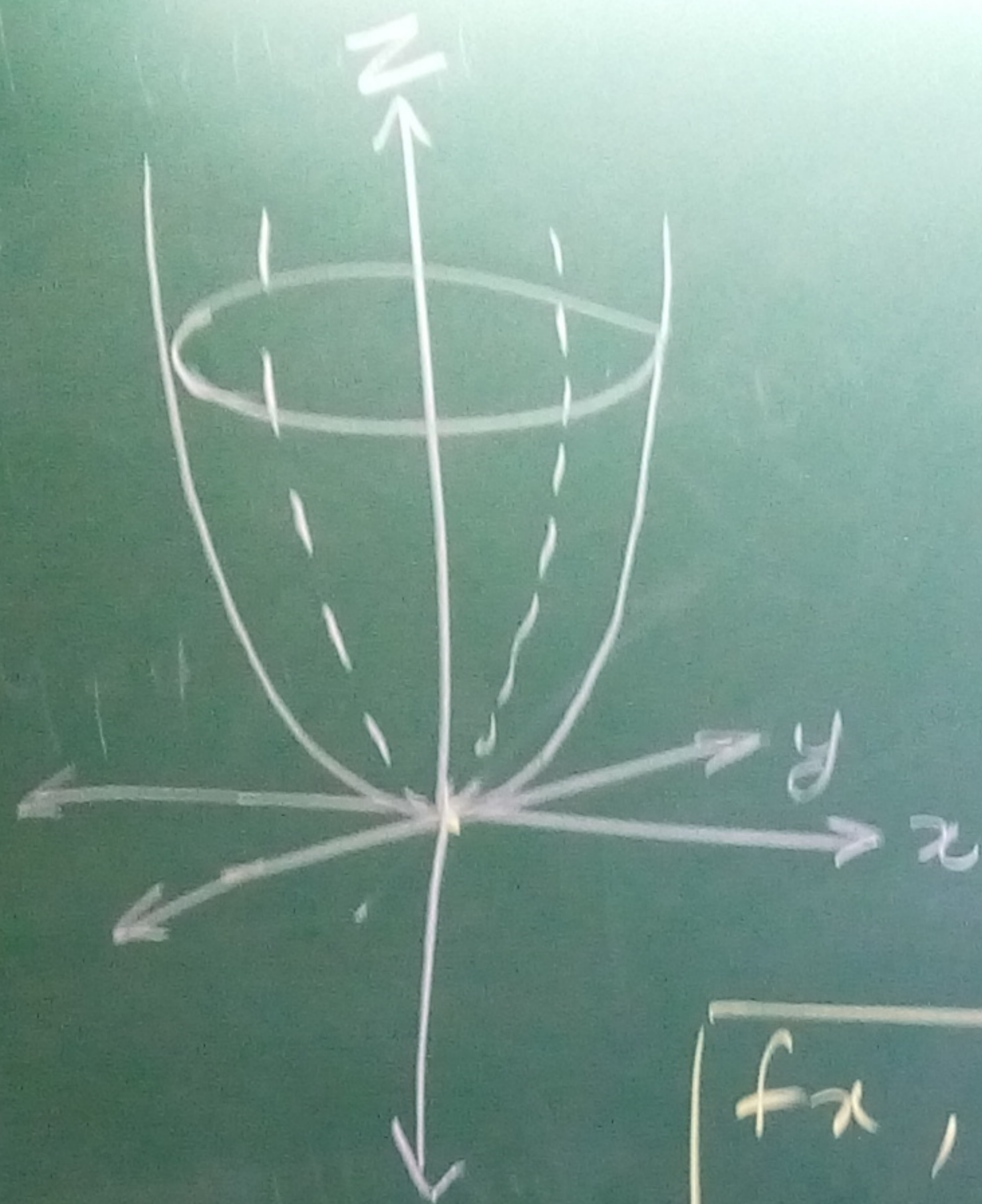
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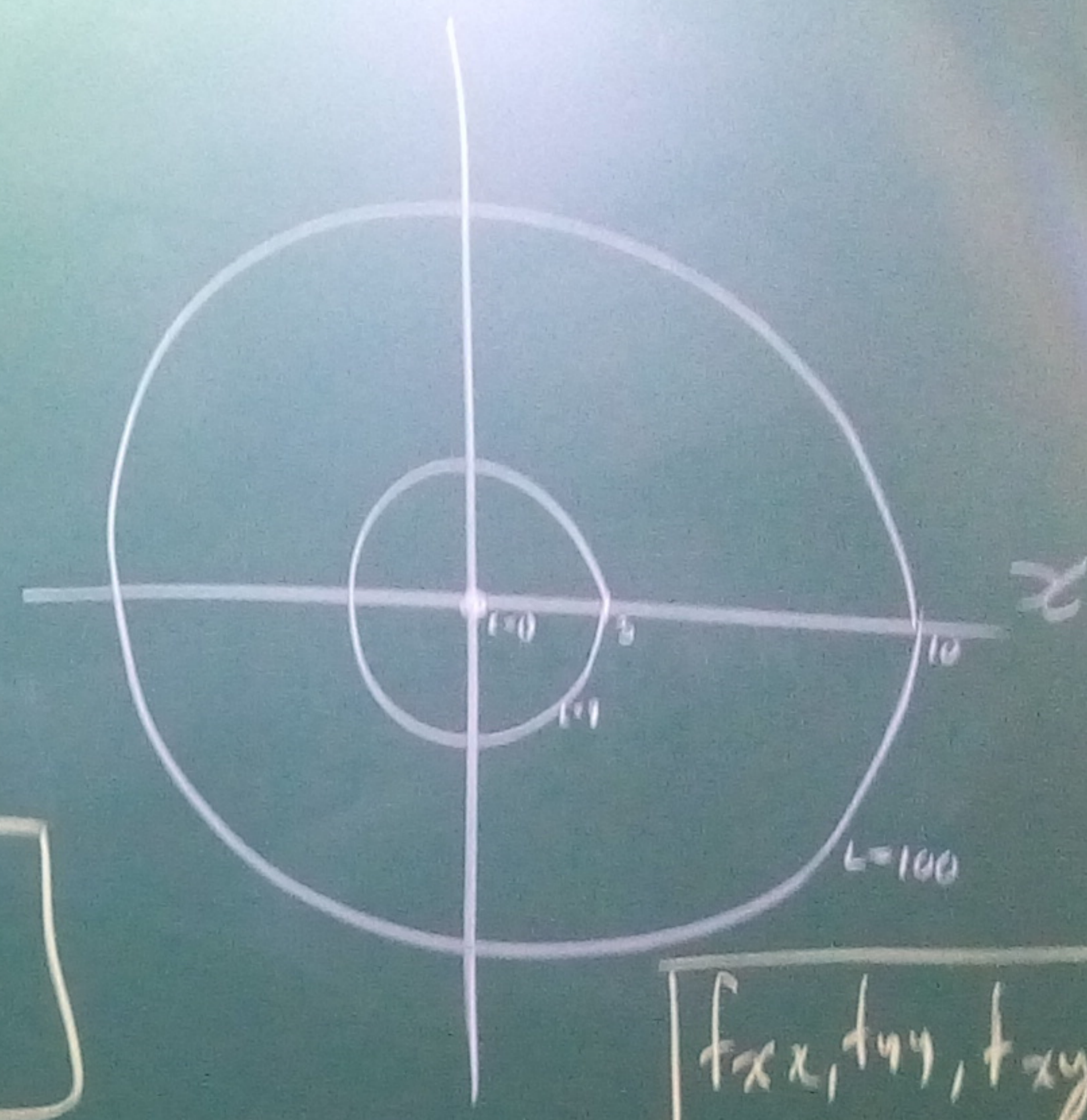
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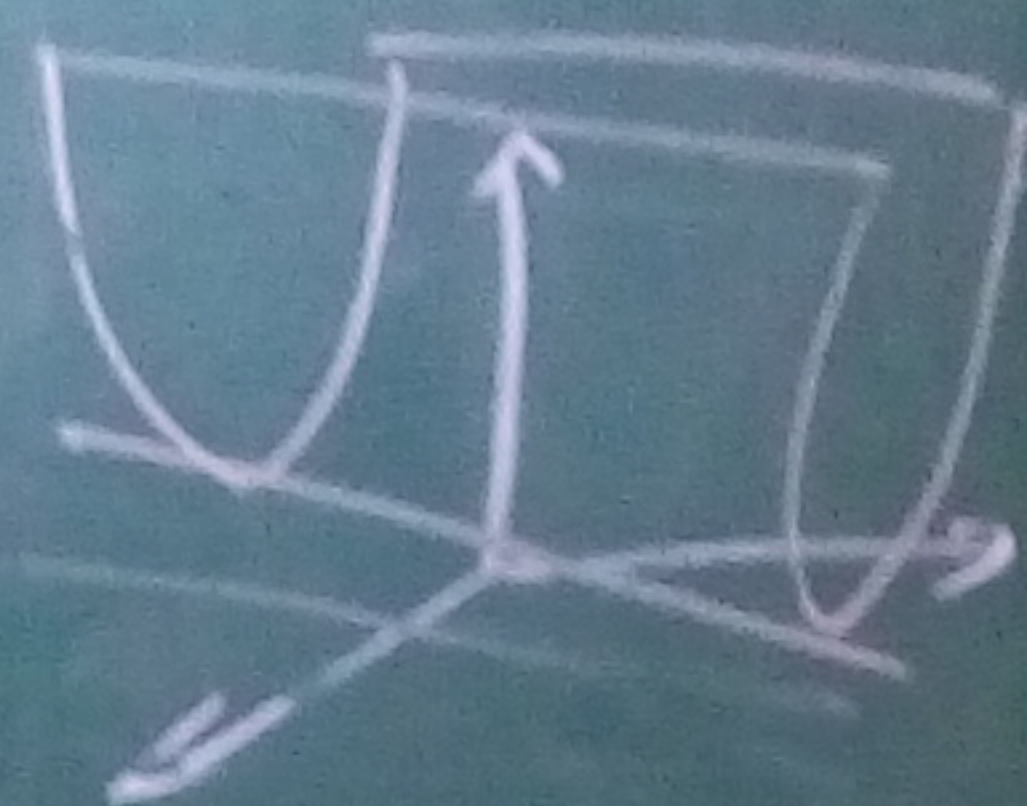
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 $h(x,y) \leq 1$

⑦ Cont. A

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$f(x,y) =$

② $a > 0$, min

$(x,y) = x + y$

③ $a < 0$

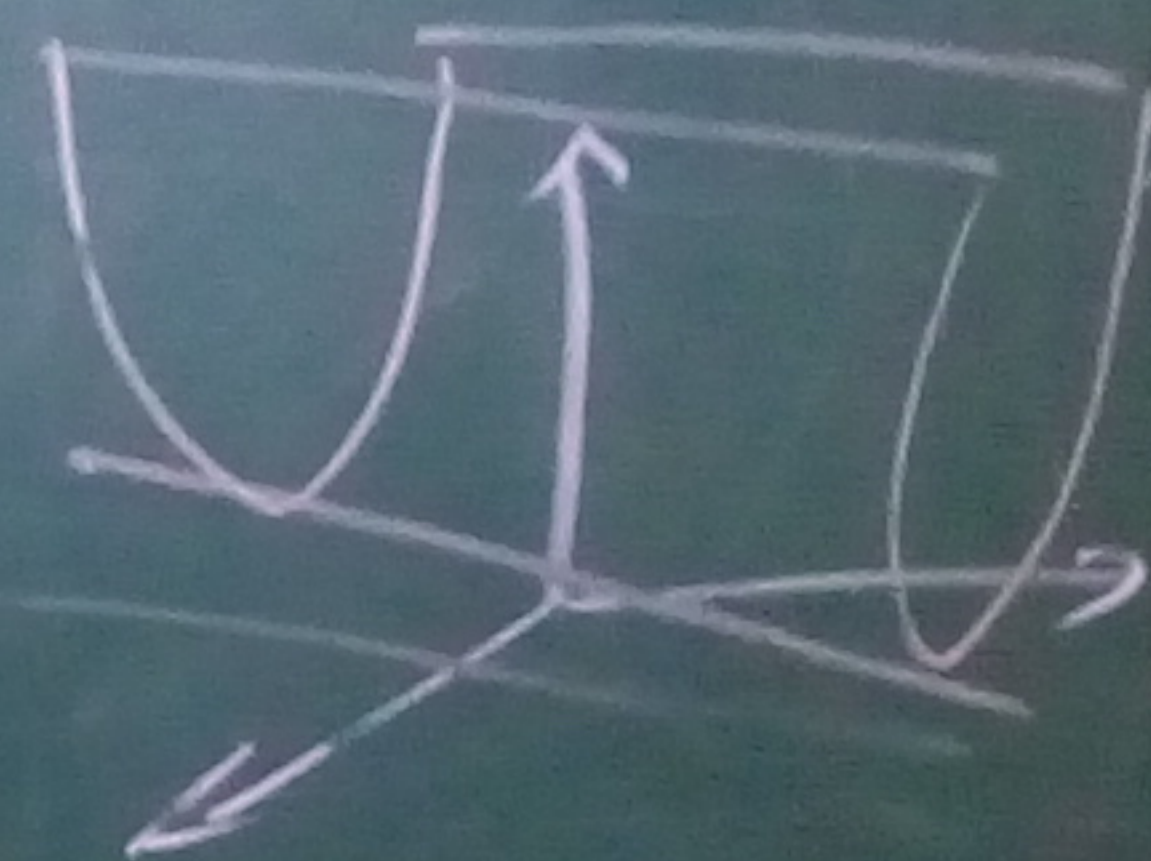
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