(a) $\forall \varepsilon > 0 \ \exists \delta > 0$ such that

for all $x \in \mathbb{R}$

$|x| < \delta \implies |f(x)| < \varepsilon$

$\boxed{(a) \iff (1)}$

(b) $\forall \delta > 0 \ \exists \varepsilon > 0$ such that

for all $x \in \mathbb{R}$

$|x| < \delta \implies |f(x)| < \varepsilon$

Result

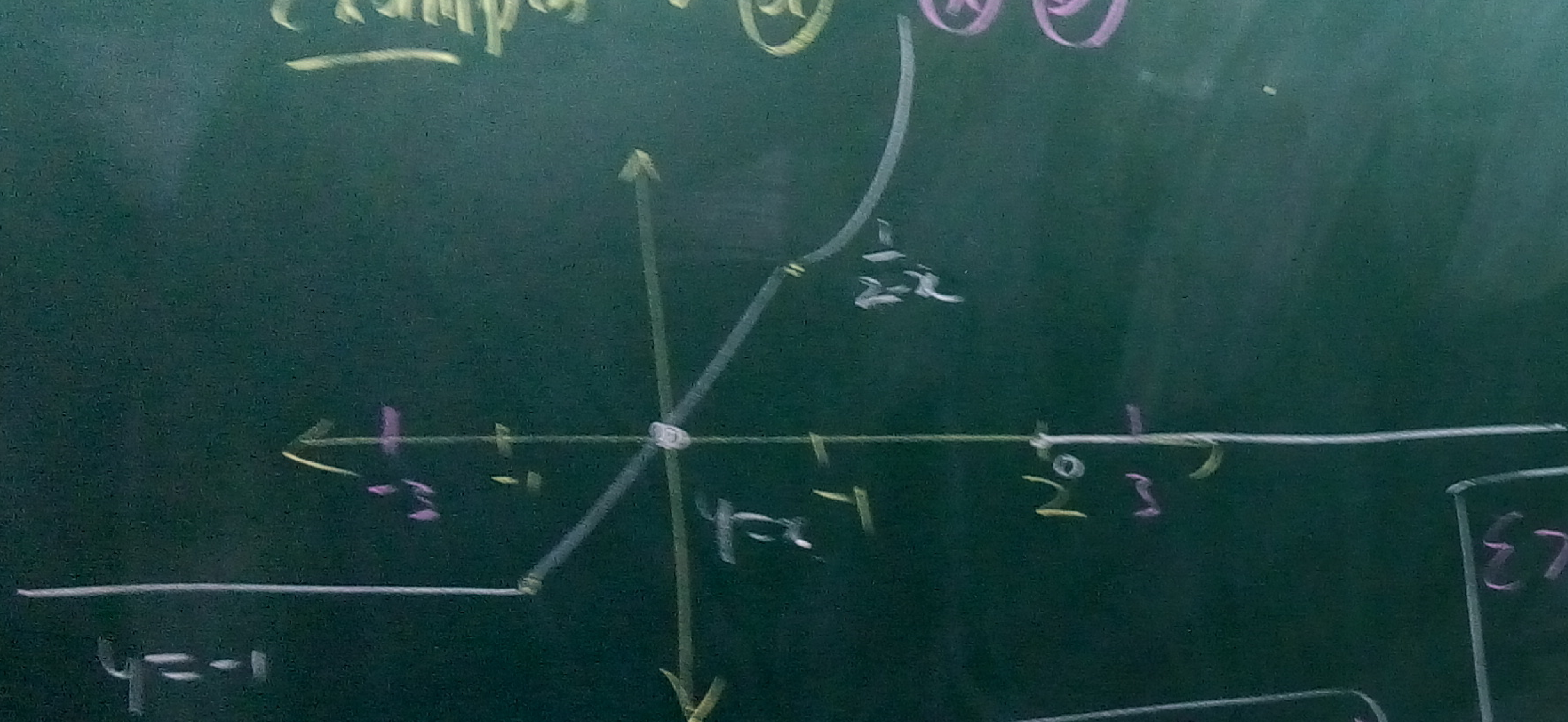
Ex

$y = -1$

$\boxed{\delta = 3}$

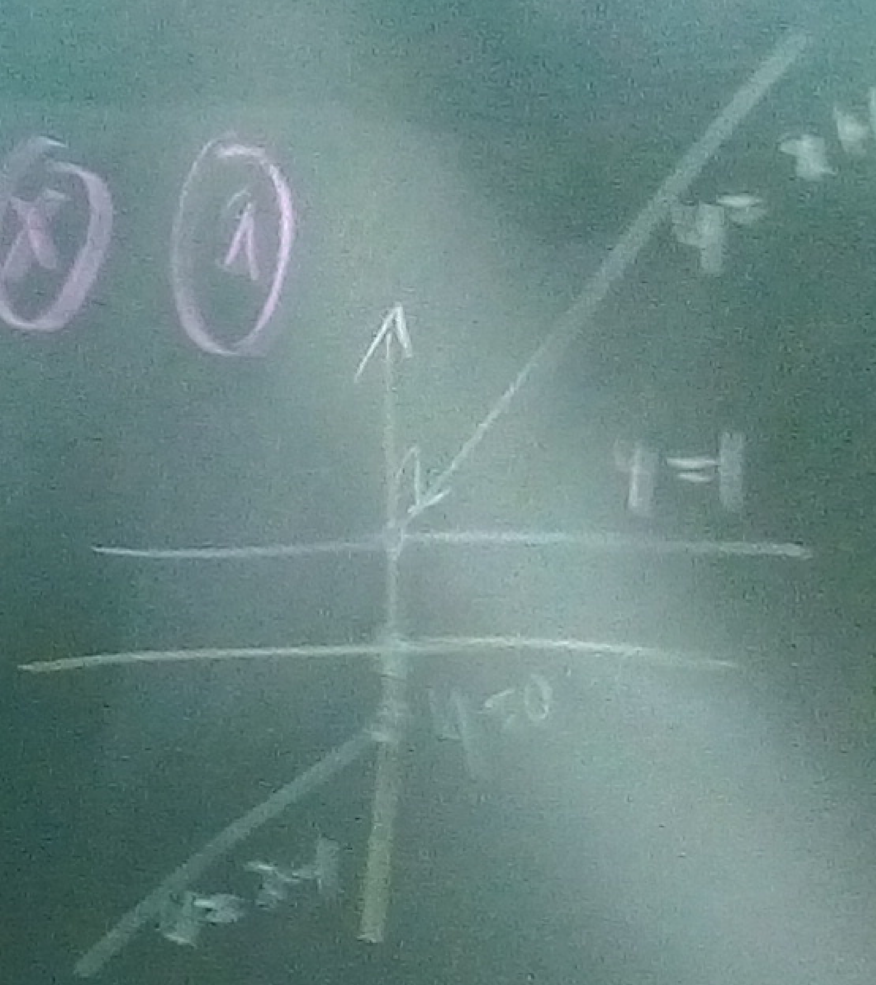
Result: (a) $\xRightarrow{\times}$ (b) & (b) $\xRightarrow{\times}$ (a)

Example ✓ (a) (x) (b) (b) ✓ (x) (a)



$\delta = 3$ As f is unbounded in $(-3,3)$ so (b) (x) hold.

$\varepsilon > 0$ $\delta = \min(\varepsilon, 1)$
(a) ✓



$\varepsilon = 2$
(b) ✓

Result $\textcircled{c} \overset{\checkmark}{\Longrightarrow} \textcircled{a}$

$\textcircled{a} \overset{\times}{\Longrightarrow} \textcircled{c}$

$\exists\, \delta > 0$ such that $\forall \varepsilon > 0$ & for all $x \in \mathbb{R}$

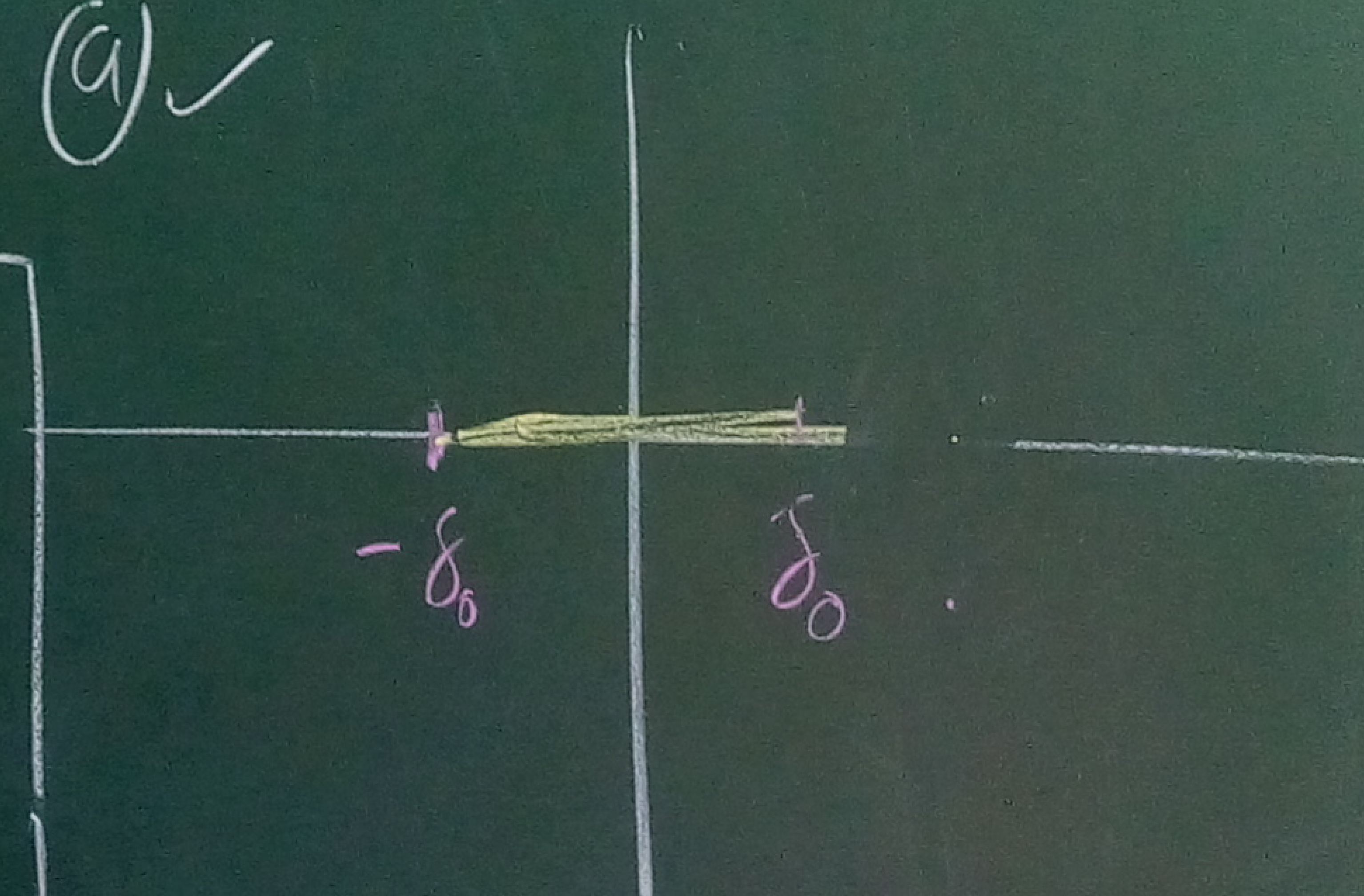
$$|x| < \delta \Longrightarrow |f(x)| < \varepsilon .$$

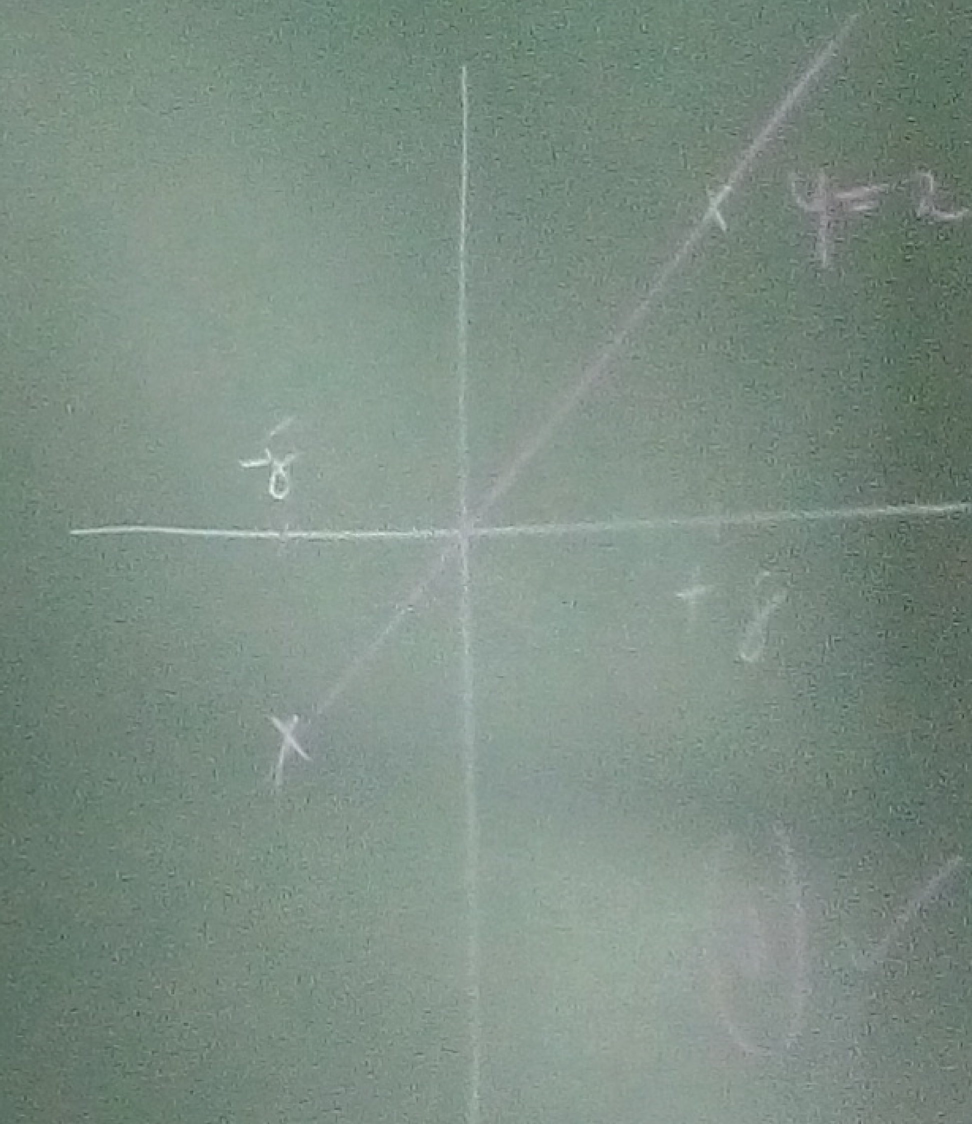
$\textcircled{c}\ \checkmark \Rightarrow \textcircled{a}\ \checkmark$

$\textcircled{d}$ $\forall \varepsilon > 0$ and $\forall x \in \mathbb{R}$

$\exists\, \delta > 0$ such that $|x| < \delta$

$$\Longrightarrow |f(x)| < \varepsilon$$





$f : \mathbb{R} \to \mathbb{R}$

$f(0) = 0$

$\varepsilon > 0,\ x \in \mathbb{R}$

$$\delta = \begin{cases} \frac{|x|}{2} & x \neq 0 \\ 1 & x = 0 \end{cases}$$