$\lim_{n\to\infty} |f(a_n)| = \infty$

3 (a)

(2) A sequence is a $f : \mathbb{N} \to \mathbb{R}$

We denote $a_n := f(n)$

In short we will call $\boxed{\{a_n\}_{n \geq 1}}$

Negation

All boys are wearing pink shirt

There exist boy who is NOT wearing pink shirt

(x) (?) ← All boys are NOT wearing pink shirt

$a_n =$

$a_n =$

$\exists M>0$

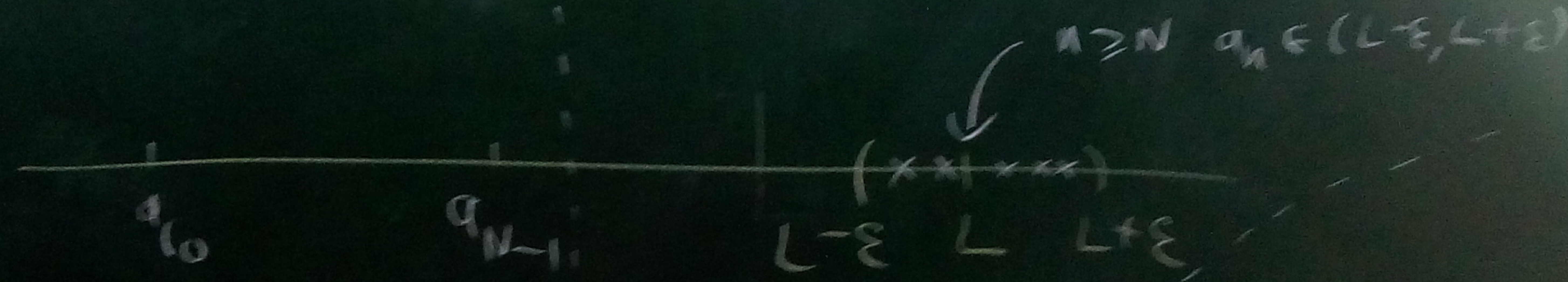
$a_n =$

a_n

3(a)

- For every $\varepsilon > 0$ there exists $N > 0$ such that $|a_n - L| < \varepsilon$ whenever $n \geq N$.

- $\forall\, \varepsilon > 0 \;\; \exists\, N_0 > 0$ such that $|a_n - L| < \varepsilon,\ \forall\, n \geq N_0$



$\exists\, \varepsilon_0 > 0,\ \forall\, N > 0 \;\; \exists\, n_0 \geq N$ such that $|a_{n_0} - L| \geq \varepsilon_0$

$a_n = c$

$a_n = \frac{(-1)^n}{n} b_n$

$\exists M > 0.\ |b_n| < M \quad \forall n \geq N$

$a_n = (-1)^n n$

$a_n = (-1)^n$

Order of Quantifiers

(P) — $\forall x \in \mathbb{R}$ $\exists y \in \mathbb{R}$ such that $x = y^3$

$xy = 0$

$(\tilde{P})$

($\because$ $A = \{ y \in \mathbb{R} \mid y^3 < x \}$

show: - A is bounded above

- let $\alpha \in \mathbb{R}$ be supremum of A
- $\alpha^3 = x$)

(Q) — $\exists y \in \mathbb{R}$ $\forall x \in \mathbb{R}$. $x = y^3$

$xy = 0$

$(\tilde{Q})$

(P) - True but Q is false

$(\tilde{P})$ is true & $(\tilde{Q})$ is true