ERROR(S)

X & Y are two non-empty sets, we define the expression.

(a) $\mathrm{Card}(X) \leq \mathrm{Card}(Y)$ if $\exists f : X \to Y$ such that f is injective

(b) $\mathrm{Card}(X) = \mathrm{Card}(Y)$ if $\exists f : X \to Y$ such that f is bijective

(c) $\mathrm{Card}(X) \geq \mathrm{Card}(Y)$ if $\exists f : X \to Y$ such that f is surjective

$\mathrm{Card}(X) < \mathrm{Card}(Y)$ if $\exists f : X \to Y$ such that f is injective & there is no bijection $g : X \to Y$

let f be increasing..

let $x, y \in \mathbb{R}$ & $f(x) = f(y)$, & $x \neq y$. — (I)

since exactly one of $x = y$, $x < y$, $x > y$ holds $(\forall x, y \in \mathbb{R})$

by (I), either $f(x) < f(y)$ or $f(x) > f(y)$ ($\because x \neq y$, & f is increasing)

Since, $f(x) < f(y)$ or $f(y) < f(x)$, $f(x) \neq f(y)$

contradicting (I)

$\therefore x \neq y$ is not possible.

So we must have $x = y$

$\therefore$ we get, $f(x) = f(y) \Longrightarrow x = y$

So f is injective

$\therefore$ f is increasing $\Longrightarrow$ f is injective.

ERRORS (Card) :-

Result 2 : For any X, Y nonem[pty]

$\text{Card}(X) \leq \text{Card}(Y)$ or

$\text{Card}(Y) \leq \text{Card}(X)$

Proof (Sketch) :

- $\mathcal{A} = \{f : A \to Y \mid A \subseteq X$ & f is injective

"Think of "I $\subseteq$ "$X \times Y$" [Definition of function]

member of

- $(I, \leq)$. partial order given by subset inclusion

  - Zorn's lemma provides a maximal element $f \in I$ & $f: A \to Y$

- let $B = \{y \in Y \mid \exists\, x \in A$ such that $f(x) = y\}$

  clearly $f: A \to B$ is bijective

If $A \neq X$ & $B \neq Y$ then $x_0 \notin A$ & $y_0 \notin B$ define $\tilde{f}: A \cup \{x_0\} \to B \cup \{y_0\}$

by $\tilde{f}(x) = \begin{cases} f(x) & x \in A \\ y_0 & x = x_0 \end{cases} \Rightarrow \tilde{f} \in I$ & $\boxed{f \leq \tilde{f}}$ contradiction