

GROUND RULES

- Siva

- On time

- Tue } 9:10 - 9:55
Thu }

- Wed 11:20 - 12:10
12:15 - 1:00 PM

Course demands careful thinking
- Emphasize writing & language skills = clarity in thinking

• Proof = a complete explanation of why something is true.

Don't get discouraged

- try to find many proofs of a single result
- clear & accurate

How to improve Writing.

• Good writing requires practice

• Proof may involve many subtleties (can be subtle)

- Expose irrelevant thoughts.

- You must write what you mean & mean what you write

- Producing a well written solution may require repeated revisions.

Q1: Babylonians . Given S and P

$$\begin{array}{l} \text{Find} \\ x, y \end{array} : \quad \begin{array}{l} x + y = S \quad - (1) \\ xy = P \quad - (2) \end{array}$$

$$\text{Equation (1)} \Rightarrow y = S - x$$

$$\text{Substitute for } y \text{ in (2)} \Rightarrow x(S - x) = P$$

$$\Rightarrow 5x - x^2 = p$$

Equivalently we need to find $x \in \mathbb{R}$ such that

$$x^2 - 5x + p = 0$$

Q2: - $a \neq 0$ $b \in \mathbb{R}$, $c \in \mathbb{R}$ Find $x \in \mathbb{R}$
Such that $ax^2 + bx + c = 0$

Ans 2:

$$0 = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

From Ans 2

If $b^2 - 4ac \geq 0$

Ans 1: The two numbers

$$\frac{5 + \sqrt{5^2 - 4p}}{2}$$

$$\frac{5 - \sqrt{5^2 - 4p}}{2}$$

$$= a \left[\left(x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a^2} \right]$$

If $b^2 - 4ac < 0$ then there
is no $x \in \mathbb{R}$ such that (LHS)
 $ax^2 + bx + c = 0$ $(\neq 0)$

$$0 = ax^2 + bx + c \quad (\Rightarrow) \quad \left(x + \frac{b}{2a} \right)^2 = \frac{b^2 - 4ac}{4a^2} \quad (*)$$

Assume $b^2 - 4ac \geq 0$ $(\Rightarrow) \quad x + \frac{b}{2a} = \sqrt{\frac{b^2 - 4ac}{4a^2}} \quad \text{or} \quad x + \frac{b}{2a} = -\sqrt{\frac{b^2 - 4ac}{4a^2}}$

$$(\Leftrightarrow) \quad x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \square$$