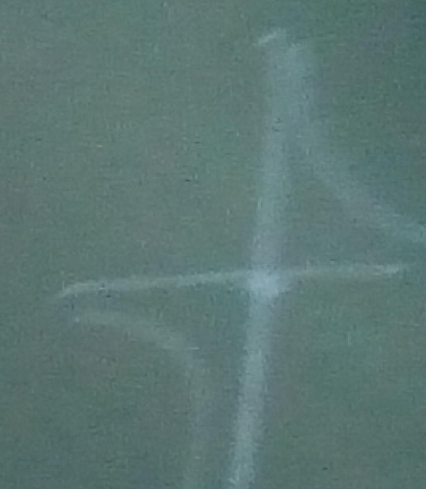
$$f(x) = \begin{cases} \dfrac{x+1}{(x-1)(x-7)} & x \notin \{1,7\} \\ 0 & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} \dfrac{1}{x} & x \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

" $\lim_{x \to 0^+} \frac{1}{x} = \infty$ " ; " $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ "



Curve sketching

① • zeros of f

$x \neq 1, 7$ $\quad \dfrac{x+1}{(x-1)(x-7)} = 0 \Rightarrow x = -1$

$f(x) = 0$ if $x = 1$, $x = 7$

• $\lim_{n \to \infty} f(1 + \frac{1}{n}) = -\infty$

• $\lim_{n \to \infty} f(1 - \frac{1}{n}) = \infty$

$f(1) = 0$

Ex

f is not continuous

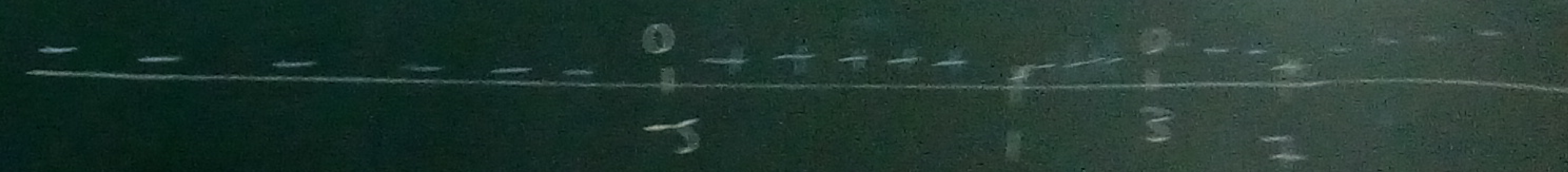
(b) Zeros of f'

Ex: Rewrite

$$f(x) = \begin{cases} \frac{4}{3}\frac{1}{(x-7)} - \frac{1}{3(x-1)} & \text{if } x \neq 1, 7 \\ 0 & \text{otherwise} \end{cases}$$

$$f'(x) = -\frac{4}{3}\frac{1}{(x-7)^2} + \frac{1}{3(x-1)^2} \qquad x \neq 1, 7, \quad \boxed{f \text{ is not diff at } 1, 7}$$

$$\overset{Ex}{=} \frac{-(x+5)(x-3)}{(x-1)^2(x-7)^2}$$



$x = -5$, $x = 3$ are zeros of f'

(c)

$$f''(x) = \begin{cases} \frac{8}{3(x-7)^3} - \frac{2}{3(x+1)^3} & x \neq 1, 7 \\ \text{Does not exist} & \text{at } x = 1, 7 \end{cases}$$

$f''(5) = \ldots$

$f''(3) = \ldots$

$f''(\ ) > 0 \quad \cup$

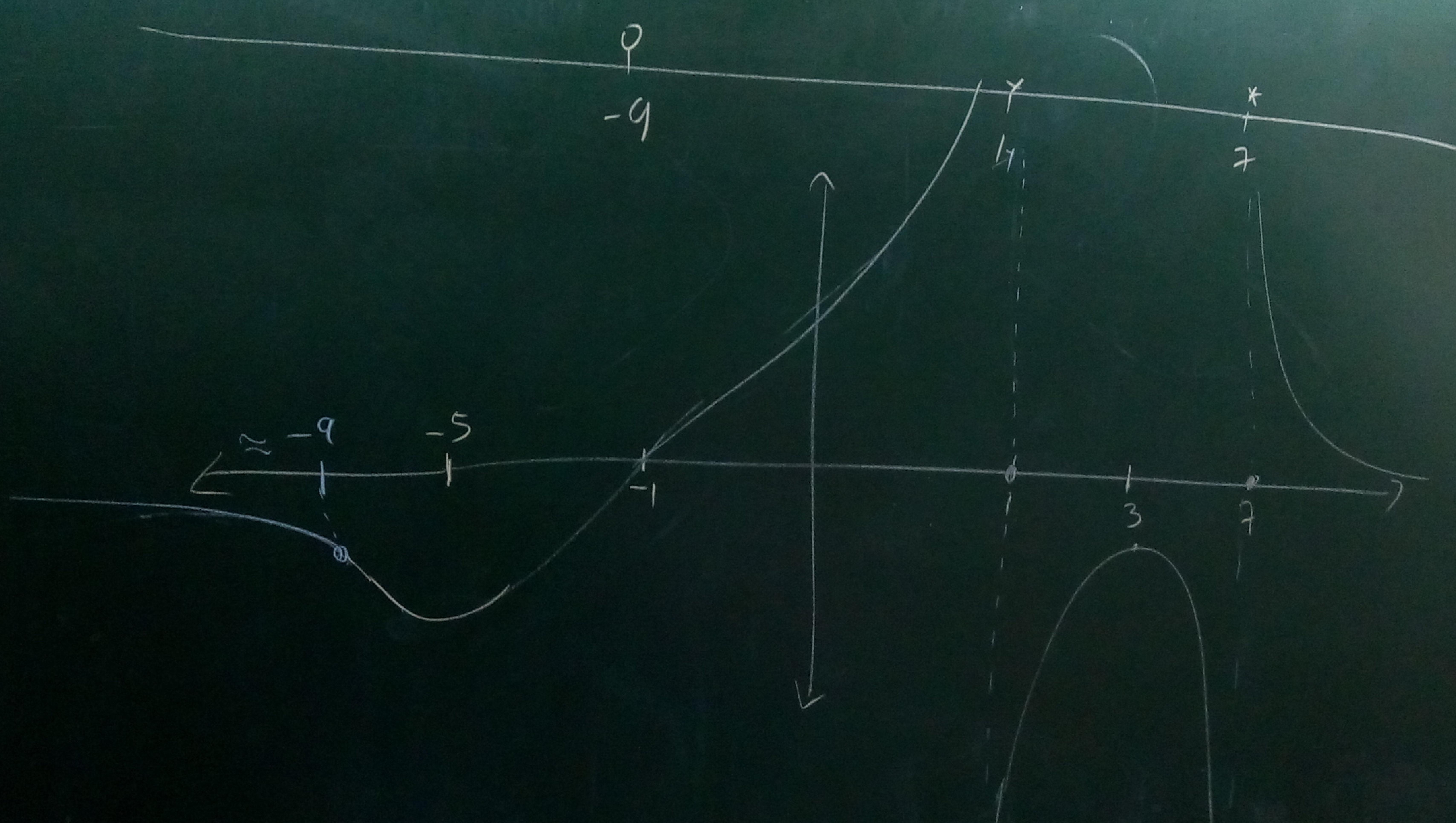
$f''(\ ) < 0 \quad \cap$

$\boxed{f(5) < 0,\ f(3) < 0 \text{ (Ex)}}$

$x \neq 1, 7$

$$f''(x) = 0 \iff \frac{8}{3(x-7)^3} = \frac{2}{3(x-1)^3} \iff \frac{4}{(x-7)^3} = \frac{1}{(x-1)^3} \iff x = -\frac{(7-4^{1/3})}{4^{1/3}-1} \simeq -9$$

Ex.-



Ex

$\lim\limits_{n \to \infty} f(1+\frac{1}{n}) = -\infty$

$\lim\limits_{n \to \infty} f(1-\frac{1}{n}) = \infty$

$\lim\limits_{n \to \infty} f(7-\frac{1}{n}) = -\infty$

$\lim\limits_{n \to \infty} f(7+\frac{1}{n}) = \infty$

$\lim\limits_{n \to \infty} f(n) = 0$

$\lim\limits_{n \to -\infty} f(n) = 0$

>0

<0

HW2 $\begin{cases} 0 < a < 1 \\ \text{"}a^2\text{"} \quad \text{prove again} \end{cases}$

$n \geq 1$, $x_n = a^{2^n}$ $\qquad$ φ: $\lim_{n \to \infty} x_n = 0$?

$\tilde{\varphi}$: $0 < a < 1$, $x_n = a^n$ $\qquad$ $\lim_{n \to \infty} x_n = 0$?

To show:

$\tilde{A}$:- $x_n \to 0$ as $n \to \infty$ $\iff$ $\lim_{n \to \infty} x_n = 0$

$\iff \forall \varepsilon > 0 \ \exists N \geq 1$ such that

$|x_n - 0| < \varepsilon$ for all $n \geq N$

(1) Let $\varepsilon > 0$ be given.

$$0 < a < 1 \implies a = \frac{1}{1+z} \text{ for some } z > 0$$

$$x_n = a^n = \left(\frac{1}{1+z}\right)^n = \frac{1}{(1+z)^n}$$

$$(1+z)^n \underset{\substack{\uparrow \\ \text{induction}}}{=} \sum_{k=0}^{n} {}^nC_k\, z^k = 1 + nz + \sum_{k=2}^{n} {}^nC_k\, z^k \underset{\boxed{z>0}}{>} 1 + nz > nz \quad \forall n \geq 1$$

$$x_n = \frac{1}{(1+z)^n} < \frac{1}{nz} \quad \forall n \geq 1 \quad \text{—} \; (**)$$

$$|x_n - 0| = |a^n - 0| = |a^n| \overset{0<a<1}{=} a^n \overset{(**)}{<} \frac{1}{n\beta}$$

<u>Archimedean</u> Property $\exists\, N \geq 1$ $N > \frac{1}{\varepsilon\beta}$ - (**) $\left[\begin{array}{c} y \in \mathbb{R}\ \&\ x>0\ \exists n \in \mathbb{N} \\ nx > y \end{array} \right]$

$$n \geq N \implies |x_n - 0| < \frac{1}{n\beta} < \frac{1}{\frac{1}{\varepsilon\beta}\beta} = \varepsilon$$

As $\varepsilon > 0$ was arbitrarily given $\implies$ $\forall \varepsilon > 0\ \exists N \geq 1$ such that $|x_n - 0| < \varepsilon \quad \forall n \geq N$.

"$\lim_{n\to\infty} x_n = 0$"