Analysis: Helley's Selection Theorem

Key words:

1. monotonically increasing function
2. pointwise convergence
3. Uniform convergence

Prove the following Theorem:

Theorem 1 *Assume that* f_n *is a sequence of monotonically increasing functions on* $\mathbb{R}$ *with* $0 \leq f_n(x) \leq 1$ *for all* x, n

(a) There is a function f *and a sequence* n_k *such that*

$$f(x) = \lim_{k \to \infty} f_{n_k}(x)$$

for all $x \in \mathbb{R}$

(b) If, f *is continuous, prove that the above convergence is uniform.*

Mathematicians in Focus: Helley and Polya