Analysis: Wierstrass Approximation Theorem

Key words:

1. Continuous Functions
2. Polynomials
3. Pointwise Convergence
4. Uniform Convergence

Solve the following Questions

1. Let $P_0 = 0$, and, define, for $n = 0, 1, 2, \ldots$ define
$$P_{n+1}(x) = P_n(x) + \frac{x^2 - P_n^2(x)}{2}$$
Prove that $\lim_{n\to\infty} P_n(x) =\mid x \mid$ uniformly on $[-1, 1]$.
2. Prove the following Theorem:

Theorem 1 *If f is a continuous real valued function on $[a, b]$, there exists a sequence of polynomials P_n such that*
$$\lim_{n\to\infty} P_n(x) = f(x)$$
uniformly on $[a, b]$.

Mathematician in Focus Weierstrass