Pyramid Algorithm

1. **Summary of 1st Stage of Pyramid Algorithm**

- transforms $\{X_t : t = 0, \ldots, N-1\}$ into wavelet & scaling coefficients
- $\frac{N}{2}$ wavelet coefficients $\{W_{1,t}, t = 0, \ldots, \frac{N}{2} - 1\}$ associated with:
  - $\mathbf{W}_1$, an $\frac{N}{2} \times 1$ vector
  - changes on scale $\tau_1 = 1$
  - first level detail $\mathcal{D}_1$
  - nominal frequencies $\frac{1}{4} \leq |f| \leq \frac{1}{2}$
  - $\mathcal{W}_1 = \mathcal{B}_1$, an $\frac{N}{2} \times N$ matrix consisting of first $\frac{N}{2}$ rows of $\mathcal{W}$
- $\frac{N}{2}$ scaling coefficients $\{V_{1,t}, t = 0, \ldots, \frac{N}{2} - 1\}$ associated with:
  - $\mathbf{V}_1$, an $\frac{N}{2} \times 1$ vector
  - averages on scale $\lambda_1 = 2$
  - first level smooth $\mathcal{S}_1$
  - nominal frequencies $0 \leq |f| \leq \frac{1}{4}$
  - $\mathcal{V}_1 = \mathcal{A}_1$, an $\frac{N}{2} \times N$ matrix spanning same subspace as last $\frac{N}{2}$ rows of $\mathcal{W}$

Please write down explicitly the elements of $\mathcal{B}_1$ and $\mathcal{A}_1$

2. **Summary of 2nd Stage of Pyramid Algorithm**

- transforms $\{V_{1,t} : t = 0, \ldots, \frac{N}{2} - 1\}$ into wavelet & scaling coefficients
- $\frac{N}{4}$ wavelet coefficients $\{W_{2,t}, t = 0, \ldots, \frac{N}{4} - 1\}$ associated with:
  - $\mathbf{W}_2$, an $\frac{N}{4} \times 1$ vector
  - changes on scale $\tau_2 = 2$
  - second level detail $\mathcal{D}_2$
  - nominal frequencies $\frac{1}{8} \leq |f| \leq \frac{1}{4}$
  - $\mathcal{W}_2 = \mathcal{B}_2\mathcal{A}_1$, an $\frac{N}{4} \times N$ matrix consisting of rows $\frac{N}{2}$ to $\frac{3N}{4} - 1$ of $\mathcal{W}$
- $\frac{N}{4}$ scaling coefficients $\{V_{2,t}, t = 0, \ldots, \frac{N}{4} - 1\}$ associated with:

- $\mathbf{V}_2$, an $\frac{N}{4} \times 1$ vector
- averages on scale $\lambda_2 = 4$
- second level smooth $\mathcal{S}_2$
- nominal frequencies $0 \leq |f| \leq \frac{1}{8}$
- $\mathcal{V}_2 = \mathcal{A}_2\mathcal{A}_1$, an $\frac{N}{4} \times N$ matrix spanning same subspace as last $\frac{N}{4}$ rows of $\mathcal{W}$

Please write down explicitly the elements of $\mathcal{B}_2$ and $\mathcal{A}_2$

3. Write down j-th stage of the Pyramid Algorithm
4. Justify that $\mathcal{W}$ so obtained is indeed an orthonormal matrix.
5. Construct your own wavelet filter and see how it does vis-a-vis the Haar wavelet.