Instructor: Siva Athreya
Time: 10:15-11:15, 11:15-12:15
Day: Friday.
Office:A12, Indian Statistical Institute, 8th Mile Mysore Road, Bangalore 560059, India.
Office Phone: 91-80-26985465
Office hours: Friday 9-10am or by appointment.

Text:
Casella G. and Berger R.I. (2002) *Statistical Inference, Duxbury Press, Belmont, California, USA*
Siva Athreya, Deepayan Sarkar and Steve Tanner, *Probability and Statistics with Examples using R*

Syllabus:

- Testing of hypotheses: Randomized and non-randomized tests. Power and size of a test. Most powerful (MP) test. NeymanPearson lemma. Uniformly most powerful (UMP) tests, Monotone likelihood ratio (MLR) property and construction of UMP tests for the families of distributions possessing MLR property, Applications. Non- existence of UMP tests for two-sided alternatives.
- 4 Similar tests and unbiased tests. Statements of theorems for the construction of UMPU tests for exponential families of distributions and their application to samples from normal distribution leading to t, χ^2 and F tests one and two-sample problems. Students t- test for correlation coefficient.

E-Mail: My e-mail address is **athreya@isibang.ac.in**. Please send me an email[1], so that I can put you on the class list. I will be emailing class announcements to this list through out the semester.

WWW-page: I shall maintain a course home-page, at the following address:
http://www.isibang.ac.in/ athreya/Teaching/st201. The page should serve as an archive.

Homework and Quizzes: There will be Homework every week and a short quiz on Friday morning.

Studying and Homework: The course requires at least two hours of study for each hour of class time. Homework will be posted on the web. It is imperative that you try, (and write up solutions for), the homework problems *BEFORE* their respective due dates. The problems given are a small selection so please do try other problems for practice from the text. You are encouraged to work together on solving the problems.

Feedback: Please feel free to drop by during office hours, to clear up difficulties or to just discuss mathematics. I would appreciate feedback from you as the course progresses. If there are suggestions/clarifications that you have then feel free to ask me.

[1] ***Free Chocolate*:** You shall get one free Chocolate, if you set-up an email account and send me email by Friday evening.

1. Let X and Y be two continuous random variables having the same distribution. Let $f :\to$ be a piecewise continuous function. Then show that $f(X)$ and $f(Y)$ have the same distribution.

2. Verify 1. for discrete random variables.

3. Let P be the empirical distribution defined by sample observations $X_1, X_2, \ldots, X_n$. In other words, P is the discrete distribution with probability mass function given by the below definition:

   Definition: Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables. The "empirical distribution" based on these is the discrete distribution with probability mass function given by

$$f(t) = \frac{1}{n}\#\{X_i = t\}.$$

   Let Y be a random variable with distribution P.

   (a) Show that $E(Y) = \bar{X}$.

   (b) Show that $Var(Y) = \frac{n-1}{n}S^2$.

4. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with finite expectation μ, finite variance σ^2, and finite $\gamma = E(X_1 - \mu)^4$. Compute $Var(S^2)$ in terms of μ, σ^2, and γ and show that $Var(S^2) \to 0$ as $n \to \infty$.

5. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with finite expectation μ and finite variance σ^2. let $S = \sqrt{S^2}$, the non-negative root of the sample variance. The quantity S is called the "sample standard deviation". Although $E[S^2] = \sigma^2$, it is not true that $E[S] = \sigma$. In other words, S is not an unbiased estimator for σ. Follow the steps below to see why.

   (a) Let Z be a random variable with finite mean and finite variance. Prove that $E[Z^2] \geq E[Z]^2$ and give an example to show that equality may not hold. (Hint: Consider how these quantities relate to the variance of Z).

   (b) Use (a) to explain why $E[S] \leq \sigma$ and give an example to show that equality may not hold.