Due : Monday, February 11th, 2002

1. ($\mathbb{R}^\infty$) Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{R}$ be the set of real numbers. Let $\mathbb{R}^\infty \equiv \mathbb{R}^\mathbb{N}$ and $\mathbb{R}^d$ be usual spaces. Let $\pi_d : \mathbb{R}^\infty \to \mathbb{R}^d$ be the usual projection map.
   (a) Define a metric d on $\mathbb{R}^\infty$ that will generate a topology equivalent to the product topology.
   (b) Show that under this topology π_d is continuous.
   (c) $A \subset \mathbb{R}^\infty$ is called a finite dimensional set if there exists d, H such that $A = \pi_d^{-1}(H)$ and $H \subset \mathbb{R}^d$. Let $\mathcal{F}$ denote the class of finite-dimensional sets. Show that $\mathcal{F}$ is a convergence determining class.

2. ($C([0,1])$) Let $C([0,1])$ be the space of continuous functions, with the uniform metric. For points $t_1, \ldots t_k \in [0,1]$, define the mapping $\pi_{t_1,\ldots t_k} : C([0,1]) \to \mathbb{R}^k$ such that

$$\pi_{t_1,\ldots t_k}(f) = (f(t_1), \ldots, f(t_k)), \quad \forall f \in C([0,1])$$

   (a) $A \subset \mathbb{R}^\infty$ is called a finite dimensional set if there exists $t_1, \ldots, t_k, H$ such that $A = \pi_{t_1,\ldots,k}^{-1}(H)$ and $H \subset C([0,1])$. Let $\mathcal{F}$ denote the class of finite-dimensional sets. Show that $\mathcal{F}$ is a determining class.
   (b) Let $n \in \mathbb{N}$. Sketch a picture of the function

$$f_n(t) = \begin{cases} nt & \text{if } 0 \le t \le \frac{1}{n} \\ 2 - nt & \text{if } \frac{1}{n} \le t \le \frac{2}{n} \\ 0 & \text{if } \frac{2}{n} \le t \le 1 \end{cases}$$

   Is there an f: such that f_n converges to f in $C([0,1])$.
   (c) Using $\mathbb{P}_n(\cdot) = \delta_{f_n}(\cdot)$. and $\mathbb{P}(\cdot) = \delta_f(\cdot)$, where $f(t) = 0$ for all t, show that $\mathcal{F}$ is not a convergence determining class.

3. If S is separable, then $\mathbb{P}_n \Rightarrow \mathbb{P}$ and $\mathbb{Q}_n \Rightarrow \mathbb{Q}$ if and only if $\mathbb{P}_n \times \mathbb{Q}_n \Rightarrow \mathbb{P} \times \mathbb{Q}$. (use Problem 3 of Homework 3- argument is also laid out in Billingsley page 20/21)

4. State Helley's selection theorem.