Due : Thursday, February 7th, 2002

1. If S is a separable and complete metric space, then each probability measure on $(S, \mathcal{B}(S))$ is tight.

2. Show that $\mathbb{P}_n \Rightarrow \mathbb{P}$ if and only if $\lim_n \int f d\mathbb{P}_n = \int f d\mathbb{P}$ for real valued bounded uniformly continuous functions.

3. Let $\mathcal{U}$ be a subclass of $\mathcal{B}(S)$ such that

   (a) $\mathcal{U}$ is closed under finite intersections

   (b) Each open set in S is a countable union of elements of $\mathcal{U}$.

   If $\mathbb{P}_n(A) \to \mathbb{P}(A)$ then $\forall A \in \mathcal{U}$ then $\mathbb{P}_n \Rightarrow \mathbb{P}$

4. Let S be a metric space. $\mathcal{V} \subset \mathcal{B}(S)$ is called a determining class if $\mathbb{P}(A) = \mathbb{Q}(A) \forall A \in \mathcal{V}$ implies that $\mathbb{P} \equiv \mathbb{Q}$. $\mathcal{W} \subset \mathcal{B}(S)$ a convergence determining class if $\mathbb{P}_n(A) \to \mathbb{P}(A) \quad \forall A \in \mathcal{V}$ implies that $\mathbb{P}_n \Rightarrow \mathbb{P}$

   (a) Give an example of a convergence determining class.

   (b) Give an example of a determining class.

   (c) Show that a convergence determining class is also a determining class.

   (d) Let $S = [0, 1)$ with the usual metric. Let $\mathcal{V} = \{[a, b) : 0 < a < b < 1\}$. Show that $\mathcal{V}$ is a determining class but not a convergence determining class.

5. Give an example

   (a) of a metric space S, probabilities $\mathbb{P}_n, \mathbb{P}$, and a function f which is bounded but not continuous such that: $\mathbb{P}_n \Rightarrow \mathbb{P}$ and $\int f d\mathbb{P}_n \not\to \int f d\mathbb{P}$

   (b) of a metric space S, probabilities $\mathbb{P}_n, \mathbb{P}$,and a function f which is $L_1(\mathbb{P})$, continuous but not bounded such that: $\mathbb{P}_n \Rightarrow \mathbb{P}$ and $\int f d\mathbb{P}_n \not\to \int f d\mathbb{P}$