Due: Wednesday Nov 9th, 2016

1. Let $X \sim$ Normal$(0,1)$. Use the moment generating function of X to calcluate $E[X^4]$.
2. Let $Y \sim$ Exponential(λ).
   (a) Calculate the moment generating function $M_Y(t)$.
   (b) Use (a) to calculate $E[Y^3]$ and $E[Y^4]$, the third and fourth moments of an exponential distriubtion.
3. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables.
   (a) Let $Y = X_1 + \cdots + X_n$. Prove that $M_Y(t) = [M_{X_1}(t)]^n$.
   (b) Let $Z = (X_1 + \cdots + X_n)/n$. Prove that $M_Z(t) = [M_{X_1}(\frac{t}{n})]^n$.
4. Suppose X is a discrete random variable and $D = \{t \in \mathbb{R} : E[t^X] \text{ exists}\}$. The function $\psi : D \to \mathbb{R}$ given by
$$\psi(t) = E[t^X],$$
is called the probability generating function for X. Calculate the probability generating function of X when X is
   (a) $X \sim$ Bernoulli(p), with $0 < p < 1$.
   (b) $X \sim$ Binomial(n,p), with $0 < p < 1$, $n \geq 1$.
   (c) $X \sim$ Geometric(p), with $0 < p < 1$.
   (d) $X \sim$ Poisson (λ), with $0 < \lambda$.
5. Let $X, X_1, X_2, \cdots, X_n$ be i.i.d random variables that are uniformly distributed over the interval $(0,1)$. Consider the first order statistic $X_{(1)} = \max\{X_1, \cdots, X_n\}$. Show that $X_{(1)}$ converges to 0 in probability.
6. Let $X_1, X_2, \ldots, X_n, \ldots$ be i.i.d. random variables with finite mean and variance. Define
$$Y_n = \frac{2}{n(n+1)} \sum_{i=1}^{n} i X_i.$$
Show that $Y_n \xrightarrow{p} E(X_1)$ as $n \to \infty$.