Due: Thursday, March 17th 2016
Problem to be turned in:

1. Suppose that $X_n \sim$ Cauchy(1) for each $n \in \mathbb{N}$ and are mutually indepenent. Find the distribution of $\bar{X}_n$.
2. Suppose that $X_n \sim$ Uniform $\{\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, 1\}$ for each $n \in \mathbb{N}$. Show that X_n converges in distribution to X as $n \to \infty$ with $X \sim$ Uniform $(0, 1)$.
3. Suppose X_n converges to X in probability, that is for any $\epsilon > 0$,
$$\lim_{n\to\infty} P(\mid X_n - X \mid > \epsilon) = 0.$$
then show that X_n converges in distribution to X.
4. Let $X \sim$ Bernoulli $(\frac{1}{2})$. Let $X_n = 1 - X$ for all $n \geq 1$. Show that X_n converges in distribution to X but X_n does not converge in probability to X.
5. (Matching problem) Consider a sample, $X_1, X_2, \ldots, X_n$ of size n, drawn from the set $\{1, 2, 3, \ldots, n\}$ without replacement. We will say that a match occurs at i, if $X_i = i$. Let N_n be the total number of matchesthat occur in such a sample.
   (a) Find the distribution of N_n.
   (b) Does N_n converge in distribution to a random variable ?
6. For $i \geq 1$, let $X_i \sim$ Exponential (1). Let $Y_n = X_{(n)} - \ln(n)$. Show that Y_n converges in distribution to a random variable X and find the distribution of X.
7. For $i \geq 1$, let $X_i \sim$ Pareto (i). Let $Y_n = nX_n - n$. Show that Y_n converges in distribution to a random variable X and find the distribution of X.