Due: Thursday, February 11th 2016
Problem to be turned in: 2,7

1. Consider the set $D = [-1, 1] \times [-1, 1]$. Let

$$L = \{(x, y) \in D : x = 0 \text{ or } \text{ or } x = -1 \text{ or } x = 1 \text{ or } y = 0 \text{ or } y = 1 \text{ or } y = -1\}$$

be the lines that create a tiling of D. Suppose we drop a coin of radius R at a uniformly chosen point in D what is the probability that it will intersect the set L ?

2. Suppose that X and Y are random variables with joint probability density

$$f(x, y) = \begin{cases} \frac{4}{5}(xy + 1) & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Are X and Y independent?
   (b) Compute the marginal densities $f_1(x)$ and $f_2(y)$ and the conditional density $f_{X|Y=y}(x)$ (for appropriate y).
   (c) Calculate the means μ_X, μ_Y, the variances σ_X^2, σ_Y^2 and the covariance σ_{XY}.
   (d) Calculate $E(X^2 + Y^2)$.

3. Let X and Y be two independent exponential random variables each with mean 1.
   (a) Find the density of $U_1 = X^{\frac{1}{2}}$.
   (b) Find the density of $U_2 = X + Y + 1$.
   (c) Find $P(\max\{X, Y\} > 1)$.

4. Let X and Y be two independent uniform $(0, 1)$ random variables. Let $U_1 = \max(X, Y)$ and $U_2 = \min(X, Y)$. Find the mean and variance of $U_1 - U_2$

5. Let $X \sim \text{Gamma}(\alpha, \lambda)$ and $Y \sim \text{Gamma } (\beta, \lambda)$. Set $Z = X + Y$. Find the conditional expectation of X given Z.

6. Let X and Y be random variables having mean 0, variance 1 and correlation ρ. Let $Z = X - \rho Y$. Find the mean and variance of Z. Find the correlation between Y and Z.

7. Let X and Y be two independent Normal $(0, 1)$ random variables. Find the distribution of $\frac{X}{Y}$. Is $\frac{X}{Y}$ independent of $X^2 + Y^2$.