

Due: Thursday, January 21st 2016

Problem to be turned in: 3,4

1. Suppose X is a uniform random variable in the interval $(0, 1)$ and Y is an independent exponential(2) random variable. Find the distribution of $Z = X + Y$.
2. Sunita makes cuts at two points selected at random on a piece of lumber of length L . Find the distribution of M , the length of the middle piece. What is the expected value of the length of the middle piece?

3. Let

$$f(x, y) = \begin{cases} \eta(x - y)^\gamma & \text{if } 0 \leq x < y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) For what values of γ can η be chosen so that f be a joint probability density function of X, Y .
 - (b) In cases as in (a), what are the values of η ?
 - (c) In such cases as in (a) and (b)
 - i. Find the marginal densities of X and Y .
 - ii. Find the distribution of $X + Y$.
4. Let X_1, X_2 be independent and identically distributed $N(0, 1)$ random variables.
 - (a) Find the distribution of $\frac{1}{2}(X_1 + X_2)$
 - (b) Find the distribution of $Y_i = X_i^2$ for $i = 1, 2$.
 - (c) Find the distribution of $Y_1 + Y_2$.
 5. Let $D = \{(x, y) : x^2 \leq y \leq x\}$. A point (X, Y) is chosen uniformly from D . Find the joint probability density function of X and Y .
 6. Let k be a positive number. Consider the joint p.d.f of X_1 and X_2 to be given by

$$f(x_1, x_2) = \begin{cases} k & \text{if } 0 \leq x_1 \leq 2, 0 \leq x_2 \leq 1 \text{ and } 2x_2 \leq x_1 \\ 0 & \text{otherwise} \end{cases}$$

Let $U = X_1 - X_2$. Find the probability density function for U .

7. Suppandi and Meera plan to meet at Gopalan Arcade between 7pm and 8pm. They decide to reach at a time (independent of each other) uniformly between 7pm and 8pm and wait for 15 minutes for the other person. Find the probability that they will meet ?