Due: Thursday, January 14th 2016
Problem to be turned in: 3.

1. Let (X, Y) be random variables whose probability density function is given by $f : \mathbb{R}^2 \to \mathbb{R}$. Find the probability density function of X and probability density function of X in each of the following cases:-
   (a) $f(x, y) = (x + y)$ if $0 \leq x \leq 1, 0 \leq y \leq 1$ and 0 otherwise
   (b) $f(x, y) = 2(x + y)$ if $0 \leq x \leq y \leq 1$ and 0 otherwise
   (c) $f(x, y) = 6x^2y$ if $0 \leq x \leq 1, 0 \leq y \leq 1$ and 0 otherwise
   (d) $f(x, y) = 15x^2y$ if $0 \leq x \leq y \leq 1$ and 0 otherwise

2. Suppose $g : \mathbb{R} \to \mathbb{R}$ be continuous and a probability density function, such that $g(x) = 0$ when $x \notin [0, 1]$. Let $D \subset \mathbb{R}^2$ be given by
$$D = \{(x, y) : x \in \mathbb{R} \text{ and } 0 \leq y \leq g(x)\}$$
Let (X, Y) be uniformly distributed on D. Find the probability density function of X.

3. Let $c > 0$. Suppose that X and Y are random variables with joint probability density
$$f(x, y) = \begin{cases} c(xy + 1) & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$
   (a) Find c.
   (b) Compute the marginal densities $f_X(x)$ and $f_Y(y)$ and the conditional density $f_{X|Y}(x|y)$

4. Suppose X is a random variable with density
$$f(x) = \begin{cases} cx^2(1 - x) & \text{for } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$
Find:
   (a) the value of c.
   (b) the distribution function of X.
   (c) the conditional probability $P(X > 0.2 \mid X < 0.5)$.

5. Continuous random variables X and Y have a joint density
$$f(x, y) = \begin{cases} k, & \text{for } 0 < x < 6, 0 < y < 4 \\ 0, & \text{elsewhere.} \end{cases}$$
   (a) Find k.
   (b) Find $P(2Y > X)$.
   (c) Find the marginal density of X.
   (d) Find the conditional density of Y given $X = 2$.
   (e) Are X and Y independent?