Due: Thursday, October 1st, 2015
Problem to be turned in: 3, 5

1. Let X and Y be discrete random variables. Let x be in the range of X and let y be in the range of Y.
   (a) Suppose X and Y are independent. Show that $E[X|Y = y] = E[X]$ (and so $E[X|Y] = E[X]$).
   (b) Show that $E[X|X = x] = x$ (and so $E[X|X] = X$).
   (c) When X and Y are independent, show that $E[X|Y]$ is a constant random variable $E[X]$.
2. Consdier the experiment of flipping two coins. Let X be the number of heads among the coins and let Y be the number of tails among the coins.
   (a) Should you expect X and Y to be posivitely correlated, negatively correlated, or uncorrelated? Why?
   (b) Calculate $Cov[X, Y]$ to confirm your answer to (a).
3. Let $X \sim$ Uniform($\{0, 1, 2\}$) and let Y be the number of heads in X flips of a coin.
   (a) Should you expect X and Y to be positively correlated, negatively correlated, or uncorrelated? Why?
   (b) Calculate $Cov[X, Y]$ to confirm your answer to (a).
4. Prove that the inequality Theorem 4.5.7. is an equality if and only if there are $a, b \in \mathbb{R}$ with $a \neq 0$ for which $P(Y = aX + b) = 1$. (Put another way, the correlation of X and Y is ± 1 exactly when Y can be expressed as a non-trivial linear function of X).
5. In class it was shown that if X and Y are independent, then $Var[X + Y] = Var[X] + Var[Y]$. If X and Y are dependent, the result is typically not true, but the covariance provides a way relate the variances of X and Y to the variance of their sum.
   (a) Show that for any discrete random variables X and Y,
   $$Var[X + Y] = Var[X] + Var[Y] + 2Cov[X, Y].$$
   (b) Use (a) to conclude that when X and Y are positively correlated, then $Var[X+Y] > Var[X]+Var[Y]$, while when X and Y are negatively correlated, $Var[X + Y] < Var[X] + Var[Y]$.