Due: Thursday, August 27th, 2015
Problem to be turned in: 6,7

1. Consider the experiment of flipping a coin four times and recording the sequence of heads and tails. Let S be the sample space of all sixteen possible orderings of the results. Let X be the function on S describing the number of tails among the flips. Let Y be the function on S describing the first flip (if any) to come up tails.
   (a) Create a table as in Example 3.2.8 discussed in class, describing functions X and Y.
   (b) Use the table to calculate $P(X = 2)$.
   (c) Use the table to calculate $P(Y = 3)$.
2. A pair of fair dice are thrown. Let X represent the larger of the two values on the dice and let Y represent the smaller of the two values.
   (a) Describe S, the domain of functions X and Y. How many elements are in S?
   (b) What are the ranges of X and Y. Do X and Y have the same range? Why or why not?
   (c) Describe the distribution of X and describe the distribution of Y by finding the probability mass function of each. Is it true that $X \sim Y$?
3. A pair of fair dice are thrown. Let X represent the number of the first die and let Y represent the number of the second die.
   (a) Describe S, the domain of functions X and Y. How many elements are in S?
   (b) Describe T, the range of functions X and Y. How many elements are in T?
   (c) Describe the distribution of X and describe the distribution of Y by finding the probability mass function of each. Is it true that $X \sim Y$?
   (d) Are X and Y the same function? Why or why not?
4. Use the $\sim$ notation to classify the distributions of the random variables described by the scenarios below. For instance, if a scenario said, "let X be the number of heads in three flips of a coin" the approrpriate answer would be $X \sim \text{Binomial}(3, \frac{1}{2})$ since that describes the number of successes in three Bernoulli trials.
   (a) Let X be the number of 5's seen in four die rolls. What is the distribution of X?
   (b) Each ticket in a certain lottery has a 20% chance to be a prize-winning ticket. Let Y be the number of tickets that need to be purchased before seeing the first prize-winner. What is the distribution of Y?
   (c) A class of ten students is comprised of seven women and three men. Four students are randomly selected from the class. Let Z denote the number of men among the four randomly selected students. What is the distribution of Z?
5. An urn has four balls labeled $1, 2, 3$, and 4. A first ball is drawn and its number is denoted by X. A second ball is then drawn from the three remaining balls in the urn and its number is denoted by Y.
   (a) Calculate $P(X = 1)$.
   (b) Calculate $P(Y = 2|X = 1)$.
   (c) Calculate $P(Y = 2)$.
   (d) Calculate $P(X = 1, Y = 2)$.

(e) Are X and Y independent? Why or why not?

6. Two dice are rolled. Let X denote the sum of the dice and let Y denote the value of the first die.
   (a) Calculate $P(X = 7)$ and $P(Y = 4)$.
   (b) Calculate $P(X = 7, Y = 4)$.
   (c) Calculate $P(X = 5)$ and $P(Y = 4)$.
   (d) Calculate $P(X = 5, Y = 4)$.
   (e) Are X and Y independent? Why or why not?

7. Let X and Y be random variables with joint distribution given by the chart below.

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	1/12	0	3/12
$Y = 1$	2/12	1/12	0
$Y = 2$	3/12	1/12	1/12

   (a) Compute the marginal distributions of X and Y.
   (b) Compute the conditional distribution of X given that $Y = 2$.
   (c) Compute the conditional distribution of Y given that $X = 2$.
   (d) Carry out a computation to show that X and Y are not independent.

Probability 1 Quiz 4 Solution Semester I 2015/16

1. A fair die is rolled repeatedly.

   (a) What is the probability that the first 6 appears on the fifth roll?
   (b) What is the probability that no 6's appear in the first four rolls?
   (c) What is the probability that the second 6 appears on the fifth roll?

Solution: We think of each roll as an independent trial. Each trial is a success if 6 appears and failure otherwise.

(a) Probability that the first 6 appears on the fifth roll is the same as the value a Geometric $(\frac{1}{6})$ puts on 5, which is

$$\left(\frac{5}{6}\right)^4 \frac{1}{6}.$$

(b) Probability that the no 6 appears in the first four rolls is the same

$$\left(\frac{5}{6}\right)^4.$$

(c) Probability that the second 6 appears in the fifth roll rolls is the same as

$$\begin{aligned} & P(\text{ Exactly one six appears in first four rolls } \cap \text{ A six appears in the fifth roll }) \\ & \quad \text{by independence of each roll} \\ = & \; P(\text{ Exactly one six in first four rolls })P(\text{ Six appears in the fifth roll }) \\ = & \binom{4}{1}\frac{1}{6}\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right), \end{aligned}$$

where exactly one six in first four rolls is the same as the value a Binomial $(4, \frac{1}{6})$ puts on for 1.