

Due: Thursday, August 13th, 2015

Problem to be turned in: 4.

1. A box of 100 tomatoes contain 5 rotten ones. What is the probability that two tomatoes chosen at random, without replacement, from the box are both not rotten ?
2. A box contains M balls, of which W are white. A sample of n balls, with $n \leq W$ and $n \leq M - W$, is drawn at random and **without** replacement. Let A_j , where $j = 1, 2, \dots, n$, denote the event that the ball drawn on the j^{th} draw is white. Find $P(A_1)$, $P(A_2)$ and $P(A_3)$. Guess what $P(A_j)$ is.
3. In a test called Narco-Analysis, a "truth" serum is given to a suspect. It is known that it is 90% reliable when the person is guilty and 99% reliable when the person is innocent. In other words 10% of the guilty are judged innocent by the serum and 1% of the innocent are judged guilty. If the suspect was selected from a group of suspects of which only 5% have ever committed a crime and the serum indicates that she is guilty, what is the probability that she is innocent ?
4. You first roll a fair die, then toss as many fair coins as the number that showed on the die. Given that 5 heads are obtained, what is the probability that the die showed 5 ?
5. *Polya Urn scheme*– An urn contains b black balls and r red balls. A ball is drawn at random. The ball is replaced into the urn along with c balls of its colour and d balls of the opposite colour. Then another random ball is drawn and the procedure is repeated.
 - (a) What is the probability that the second ball drawn is a black ball ?
 - (b) Assume $c = d$. What is the probability that the second ball drawn is a black ball ?
 - (c) Assume $c = d$. What is the probability that the n^{th} ball drawn is a black ball ?
6. A box contains M balls, of which W are white. A sample of n balls is drawn at random, **with** replacement. Let A_j , where $j = 1, 2, \dots, n$, denote the event that the ball drawn on the j^{th} draw is white. Let B_k denote the event that the sample of n balls contains exactly k white balls. Find $P(A_j|B_k)$.
7. It is estimated that 0.8% of a large shipment of eggs to a certain supermarket are cracked. The eggs are packaged in cartons, each with a dozen eggs, with the cracked eggs being randomly distributed. A restaurant owner buys 10 cartons from the supermarket.
 - (a) If she notes the number of cartons containing cracked eggs, what are the possible outcomes for this experiment?
 - (b) If she notes the total number of cracked eggs, what are the possible outcomes for this experiment?
 - (c) How likely is it that she will find exactly one cracked egg among all of her cartons?
 - (d) How likely is it that exactly one of her cartons will contain at least one cracked egg?
 - (e) Explain why your answer to (d) is close to, but slightly larger than, than your answer to (c).
 - (g) What is the most likely number of cracked eggs she will find among her cartons?
 - (h) What is the most likely number of cartons that will contain at least one cracked egg?
 - (i) How do you reconcile your answers to parts (g) and (h)?