Due: Thursday, October 27th, 2015
Problem to be turned in: 3,6

1. Let X be a random variable with density $f(x) = 2x$ for $0 < x < 1$ (and $f(x) = 0$ otherwise).
   (a) Calculate $E[X]$. You should get a result larger than $\frac{1}{2}$. Explain why this should be expected even without computations.
   (b) Calculate $SD[X]$.
2. Let $X \sim$ Uniform$(0, 10)$ and let $g(x) = \max\{x, 4\}$. Calculate $E[g(X)]$.
3. Let $X \sim$ Unifrom(a, b). Let μ and σ be the expected value and standard deviation of X.
   (a) Calculate $P(|X - \mu| \leq k\sigma)$. Your final answer should depend on k, but not on the values of a or b.
   (b) What is the value of k such that results of more than k standard deviations from expected value are unachievable for X?
   (c) Repeat (a) and (b) when $X \sim$ Exponential(λ)
4. Let $r \geq 1$. Suppose we have a coin with probability of heads being p. We toss the coin till we obtain r heads. Let X be the trial at which the r-th head occurs. Find the probability and moment generating functions of X.
5. Let $X \sim$ Normal $(0, 1)$. Use the moment generating function of X to calcluate $E[X^4]$.
6. Let $Y \sim$ Exponential (λ).
   (a) Calculate the moment generating function $M_Y(t)$.
   (b) Use (a) to calculate $E[Y^3]$ and $E[Y^4]$, the third and fourth moments of an exponential distriubtion.