Due: Thursday, October 8th, 2015
Problem to be turned in: 2, 4(a)

1. Let $f(x)$ be defined by
$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
   (a) Show that $f(x)$ is a probability density function.
   (b) Let P be the probability whose density is f. Calculate $P((0, \frac{1}{2}))$.

2. Let $f(x)$ be defined by
$$f(x) = \begin{cases} C \cdot sin(x) & \text{if } 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$
   where C is a yet-to-be-determined constant.
   (a) Determine the value of C that makes $f(x)$ a probability density function.
   (b) Let P be the probability whose density if f. Calculate $P((0, \frac{1}{2}))$ and $P((\frac{1}{2}, 1))$.
   (c) Which will be larger, $P((0, \frac{1}{4}))$ or $P((\frac{1}{4}, \frac{1}{2}))$? Explain how you can answer this question without actually calculating either probability.
   (d) A game is played in the following way. A random variable X is selected with a density described by $f(x)$ above. You must select a number r and you win the game if the random variable results in an outcome in the interval $(r - 0.01, r + 0.01)$. Explain how you should choose r to maximize your chance of winning the game. (Very little computation should be required to answer this).

3. Let $X \sim \text{Exp}(\lambda)$. The "90th percentile" is a value a such that X is larger than a 90% of the time. Find the 90th percentile of X by determining the value of a for which $P(X < a) = 0.9$.

4. The "median" of a continuous random variable X is the value of x for which $P(X > x) = P(X < x) = \frac{1}{2}$.
   (a) If $X \sim \text{Uniform}(a, b)$ calculate the median of X.
   (b) If $Y \sim \text{Exp}(\lambda)$ calcluate the median of Y.
   (c) Let $Z \sim \text{Normal}(\mu, \sigma^2)$. Show that the median of Z is μ.

5. Let $X \sim \text{Normal}(\mu, \sigma^2)$. Show that $P(|X - \mu| < k\sigma)$ does not depend on the values of μ or σ.