Not Due

1. Let X_n be a simple random walk, i.e. $X_n = \sum_{i=1}^n \xi_i$ where ξ_i are i.i.d. taking values 1 and -1 with probability $\frac{1}{2}$. Now suppose, for $x \in \mathbb{Z}$ let

$$V(x) = \sum_{k=1}^{\infty} 1(X_n = x), h^{(m)}(x) = P(V(x) \geq m), m \geq 1.$$

   (a) Show that $P(\cap_{n=m+k}^{\infty} X_n \neq y) \geq P(X_k = x)P(\cap_{n=m}^{\infty} X_n \neq y)$.

   (b) Conclude from above that $h(x) > 0, h^{\infty}(y) = 1$ implies that $h^{\infty}(y - x) = 1$.

   (c) Further show that $h(x) > 0$, $h(0) = 1$ implies that $h^{\infty}(x) = 1$

   The above exercise along with the work done in class, where we showed that $h(0) = 1, h^{\infty}(0) = 1$, implies that the simple random walk visits every site infinitely often. The simple random walk is called recurrent.

2. Consider a simple random walk in $\mathbb{Z}^d$ with $d \geq 2$, i.e at each time step the walk jumps to one of its neighbours with probability $\frac{1}{2d}$. Show that the walk in 2 dimensions is recurrent but in 3 dimensions is not. *Hint: define* $g(x) = \sum_{k=1}^{\infty} P(X_k = x)$ *and decided whether it is finite or not. Then use the equivalences between* h *and* g *discussed in class.*