Due: Tuesday, January 19th, 2010

Problem to be turned in: 3.

1. A die is rolled and then a coin is tossed. Describe the sample space for this experiment.

2. Suppose A and B are subsets of a sample space Ω.

   (a) Show that $(A - B) \cup B = A$ when $B \subset A$.

   (b) Show by example that the equality doesn't always hold if B is not a subset of A.

3. Consider a deck of 50 cards. Each card has one of 5 colors (black, blue, green, red, and yellow), and is printed with a number (1,2,3,4,5,6,7,8,9, or 10) so that each of the 50 color/number combinations is represented exactly once. A **hand** is produced by dealing out five different cards from the deck. The order in which the cards were dealt does not matter.

   (a) How many different hands are there?

   (b) How many hands consist of cards of identical color?

   (c) How many hands contain exactly three cards with one number, and two cards with a different number?

   (d) How many hands contain two cards with one number, two cards with a different number, and one card of a third number?

4. Assuming the three axioms of probability show the following:

   (a) If A is an event and $\bar{A}$ its complement, then $P(\bar{A}) = 1 - P(A)$

   (b) If A, B and C are events then, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

   (c) Suppose that E and F are events.If $E \subset F$, then $P(F - E) = P(F) - P(E)$. Show that $P(E \cap F) \leq P(E \cup F) \leq P(E) + P(F)$.

5. Suppose we toss two fair dice: Let E_1 denote the event that the sum of the dice is six. E_2 denote the event that sum of the dice equals seven. Let F denote the event that the first die equals four. Is E_1 independent of F ? Is E_2 independent of F ?

6. Suppose that each of three women at a party throws here hat into the center of the room. The hats are first mixed up and then each one randomly selects a hat. What is the probability that none of the three women selects her own hat ?

7. Suppose that an airplane engine will fail, when in flight, with probability $1 - p$ independently from engine to engine; suppose that the airplane will make a successful flight if atleast 50 percent of its engines remain working. For what values of p is a four-engine plane preferable to a two-engine plane ?

8. A fair die is rolled repeatedly.

(a) What is the chance that the first 6 appears before the tenth roll.

(b) What is the chance that the third 6 appears on the tenth roll.

(c) Given that there were six 6's among the first twenty rolls, what is the chance of seeing three 6's among the first ten rolls.

9. A box contains M balls, of which W are white. A sample of n balls, with $n \leq W$ and $n \leq M - W$, is drawn at random and without replacement. Let A_j, where $j = 1, 2, \cdots, n$, denote the event that the ball drawn on the j^{th} draw is white. Find $P(A_1)$, $P(A_2)$ and $P(A_3)$. Guess what $P(A_j)$ is.

10. A box contains 500 envelopes, of which 50 contain Rs 100 in cash, 100 contain Rs 50 in cash and 350 contain Rs 10. An envelope can be purchased at Rs 25 from the owner, who will pick an envelope at random and give it to you. Write down the sample space for the net money gained by you. What is the probability that the first envelope purchased contains less than 100 Rs ?

11. Sheela has lost her key to her room. The security officer gives her 50 keys and tells her that one of them will open her room. She decides to try each key successively and notes down the number of the attempt at which the room opens. Describe the sample space for this experiment and also decide on the probability of each outcome.

12. (*Feller Volume 1 page 56, problem 20*) From a population of n elements a sample of size r is taken. Find the probability that none of the N prescribed elements will be included in the sample, assuming the sampling to be (a) without, (b) with replacement.

13. Suppose that r indistinguishable balls are placed in n distinguishable boxes so that each distinguishable arrangement is equally likely. Find the probability that no box will be empty.

14. If n balls are placed at random into n boxes, find the probability that a) none of the boxes are empty and b) that exactly one box remains empty ?

15. Suppose that n sticks are broken into two one long and one short piece. The $2n$ pieces are now arranged into n pairs from which new sticks are formed. Find the probability (a) that the pieces will be joined in the original order, (b) that all long pieces are paired with short pieces.

16. Suppose a population of size N contains G Green people and B Blue people, with $N = G + B$. A sample of size n, $1 \leq n \leq N$ is taken. Find the probability that g, $1 \leq g \leq \min(n, G)$, Green people are selected in the sample, assuming the sampling to be (a) without, (b) with replacement.