Due: Thursday, September 3rd, 2009 *Problem to be turned in: 1, 8*
Due: Tuesday, October 6th, 2009 *Problem to be turned in: 5,9*

1. Let $i, j \in \mathbb{N}$. Show that the function $p(i,j) = \frac{1}{2^{i+j}}$ is a probability mass function. Now, consider X, Y be two random variables on a probabilty space $(\Omega, \mathcal{F}, P)$ with joint probability mass function given by $p(\cdot, \cdot)$. Compute $P(X + Y = 4)$ and $P(X - Y = 2)$.

2. Let X be the minimum and Y be the maximum of three digits picked at random wthout replacement from $\{0, 1, \ldots, 9\}$. Find the joint distribution of X and Y.

3. Of the people who enter a blood bank to donate blood, 1 in 3 have type O^+ blood and 1 in 15 have type O^- blood. For the next three people entering the blood bank, let X denote the number with type O^+ blood and Y the number with type O^- blood. Find the probability distributions for X, Y and $X + Y$.

4. Suppose that the number of earthquakes that occur in a year, anywhere in the world, is a Poisson random variable with mean λ. Suppose that the probability that any given earthquake has magnitude at least 5 on the Richter scale is p. Find the probability that there are n earthquakes with magnitude at least 5 in a year.

5. The joint probability mass function $p(x, y)$ of the random variables X and Y is given by the following table. Determine if X and Y are independent.

$p(x,y)$	$y=0$	$y=1$	$y=2$	$y=3$
$x=0$	0.1681	0.1804	0.0574	0.0041
$x=1$	0.1804	0.1936	0.0616	0.0044
$x=2$	0.0574	0.0616	0.0196	0.0014
$x=3$	0.0041	0.0044	0.0014	0.0001

6. Suppose Ω is a sample space consisting of sequences of two coin flips. Let X be a r.v. that is 1 if the first coin is heads, and 0 otherwise, Y be a r.v. that is 1 if the first coin is tails, and 0 otherwise, and Z be a r.v. that is 1 if the second coin is tails, and 0 otherwise. (a) Show that X, Y, and Z all have the same p.m.f. (b) Show that the pairs (X, Y) and (X, Z) have different joint p.m.f.s. (c) Are X and Y independent? Why or why not? (d) Are X and Z independent? Why or why not?

7. Consider a sample space Ω consisting of sequences of five coin flips. Let X be a random variable equal to the number of heads that come up from the five flips.

   a) What is the range of X?

   b) What is the p.m.f. of X that turns this range into a probability space?

8. Suppose tickets numbered $\{1, 2, \ldots, n\}$ are placed in a box and drawn one by one at random without replacement. Let X_i be the number of the ith ticket drawn, $1 \leq i \leq n$. (a) Find the joint distribution of $(X_1, X_2, \ldots, X_n\}$. (b) Find the distribution X_j for $1 \leq j \leq n$.

9. Let X and Y be independent random variables each geometrically distributed with parameter p. (a) Find the distribution of $\min(X, Y)$. (b) Find $P(\min(X, Y) = X)$. (b) Find the distribution of $X + Y$. (c) Find the $P(X > m + n | X > n)$. (d) Find $P(Y = y | X + Y = z)$.

10. *Extra credit:* A box contains red and black balls. First, 10 balls are drawn with replacement and let X_1 be the number of red balls. Next, 10 balls are drawn with replacement and let X_2 be the number of red balls in this second sample. Are X_1 and X_2 independent ?