

Due: Thursday, August 27th, 2009

Problem to be turned in: 3,6,8

1. A box contains M balls, of which W are white. A sample of n balls is drawn at random, **with** replacement. Let A_j , where $j = 1, 2, \dots, n$, denote the event that the ball drawn on the j^{th} draw is white. Let B_k denote the event that the sample of n balls contains exactly k white balls. Find $P(A_j|B_k)$.
2. Two bookies, Gupta and Netika, bet Rupees 100 each on successive tosses of a coin. Each has Rupees 600 in hand.
 - (a) What is the probability that they break even after six tosses of the coin?
 - (b) What is the probability that Netika wins all of the money on the tenth toss of the coin?
3. Suppose that airplane engines operate independently in flight and fail with probability p ($0 \leq p \leq 1$). A plane makes a safe flight if at least half of its engines are running. Kingfisher Air lines has a four-engine plane and Paramount Airlines has a two-engine plane for a flight from Bangalore to Delhi. Which airline has the higher probability for a successful flight ?
4. At Indian Statistical Institute, Kolkata there are 598 students. What is the probability that more than five students were born on Independence day ? Assume that birthrates are constant throughout the year and that each year has 365 days.
5. Suppose a fair coin is tossed n times. Find
 - (a) $P(\{4 \text{ heads occur}\}|\{3 \text{ or } 4 \text{ heads occur}\})$,
 - (b) $P(\{k - 1 \text{ heads occur}\}|\{k - 1 \text{ or } k \text{ heads occur}\})$, and
 - (c) $P(\{k \text{ heads occur}\}|\{k - 1 \text{ or } k \text{ heads occur}\})$.
6. Two types of coin are produced at a factory: a fair coin and a biased one that comes up heads 55% of the time. We have one of these coins but do not know whether it is a fair or biased coin. In order to ascertain which type of coin we have, we shall perform the following statistical test. We shall toss the coin 1000 times. If the coin comes up heads 525 or more times we shall conclude that it is a biased coin. Otherwise, we shall conclude that it is fair. If the coin is actually fair, what is the probability that we shall reach a false conclusion? What would it be if the coin were biased?
7. At a Doordarshan National TV opinion poll, they wish to know the percentage p of people who intend to vote for congress. How large must a random sample with replacement be in order to be at least 95% sure that the sample percentage is within one percent of p ?
8. Suppose we perform 500 independent trials with probability of success being 0.02. Using the Poisson approximation, find the probability that there are 2 or fewer successes.