

Due: Thursday, August 20th, 2009

Problem to be turned in: 4.

1. A box contains M balls, of which W are white. A sample of n balls, with $n \leq W$ and $n \leq M - W$, is drawn at random and **without** replacement. Let A_j , where $j = 1, 2, \dots, n$, denote the event that the ball drawn on the j^{th} draw is white. Find $P(A_1)$, $P(A_2)$ and $P(A_3)$. Guess what $P(A_j)$ is.
2. In a test called Narco-Analysis, a "truth" serum is given to a suspect. It is known that it is 90% reliable when the person is guilty and 99% reliable when the person is innocent. In other words 10% of the guilty are judged innocent by the serum and 1% of the innocent are judged guilty. If the suspect was selected from a group of suspects of which only 5% have ever committed a crime and the serum indicates that she is guilty, what is the probability that she is innocent ?
3. You first roll a fair die, then toss as many fair coins as the number that showed on the die. Given that 5 heads are obtained, what is the probability that the die showed 5 ?
4. *Polya Urn scheme*– An urn contains b black balls and r red balls. A ball is drawn at random. The ball is replaced into the urn along with c balls of its colour and d balls of the opposite colour. Then another random ball is drawn and the procedure is repeated.
 - (a) What is the probability that the second ball drawn is a black ball ?
 - (b) Assume $c = d$. What is the probability that the second ball drawn is a black ball ?
 - (c) Assume $c = d$. What is the probability that the n^{th} ball drawn is a black ball ?
5. A box of 100 tomatoes contain 5 rotten ones. What is the probability that two tomatoes chosen at random, without replacement, from the box are both not rotten ?
6. Shyam is randomly selected from the citizens of Hyderabad by the Health authorities. A laboratory test on his blood sample tells Shyam that he has tested positive for Swine Flu. It is found that 95% of people with Swine Flu test positive but 2% of people without the disease will also test positive. Suppose that 1% of the population has the disease. What is the probability that Shyam indeed has the Swine Flu ?

Ahead in class:- Suppose you toss a (fair or biased) coin n times. What is the most likely number of heads that you will obtain ? On an average what is the number of heads you will obtain ?

Reference Text added: Hoel, Port, and Stone, *Introduction to Probability Theory*.