Due: January 28th, 2008
Problems to be turned in: 3,4

1. Let $b(n,p,j)$ denote the probability of getting j successes in a Binomial(n,p) experiment. Let $\phi : \mathbb{R} \to \mathbb{R}$ be given by:
$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2}), \quad -\infty < x < \infty$$
Show that
$$\lim_{n\to\infty} \sqrt{npq}\, b(n,p,[np + x\sqrt{npq}]) = \phi(x)$$

2. Suppose we conduct an experiment having two outcomes ($\{S\}$ Success happens with probability p and $\{F\}$ Failure happens with probability $1-p$ for $0 < p < 1$) n times. Let $(\Omega, \mathcal{F}, P)$ be the corresponding probability space. Define S_n to be the number of successes in n trials. Show that
$$\lim_{n\to\infty} P(a \le \frac{S_n - np}{\sqrt{npq}} \le b) = \int_a^b \phi(x)$$

3. Show that the, m, *Mode* of the Binomial(n,p) distribution is given by $m = [np + p]$. Further clarify that (depending on n,p)
   (a) if np happens to be an integer then $m = np$.
   (b) if np is not an integer then m is one of the two integers to either side of np.
   (c) m may not necessarily be closest integer to np and neither is m always the integer above np nor the integer below it.

4. An airline knows that over the long run, 90% of passengers who reserve seats show up for their flight. On a particular flight with 300 seats, the airline accepts 324 reservations.
   (a) Assuming that passengers show up independently of each other, what is the chance that the flight will be overbooked ?
   (b) Suppose that people tend to travel in groups. Would that increase of decrease the probability of overbooking ? Explain your answer.
   (c) Redo the calculation a) assuming that the passengers always travel in pairs. Check that your answers to (a), (b) and (c) are consistent.

5. Let $z > 0$. If $\Phi(z) = \int_{-\infty}^{z} \phi(x)dx$ then show that $1 - \Phi(z) \le \frac{\phi(z)}{z}$

6. Let S_{25} be the number of successes in a Binomial $(25, \frac{1}{10})$ experiment.
   (a) Find m
   (b) Find $P(S = m)$ correct upto 3 decimal places.
   (c) What is the value of the Normal approximation to $P(S = m)$?
   (d) What is the value of the Poisson approximation to $P(S = m)$?
   (e) Repeat the above if 25 is replaced by 2500. Compare the approximations given by Normal and Poisson. Repeat the same with 2500 and $\frac{1}{10}$ replaced by $\frac{1}{1000}$