Problems due: X where X is Uniform $\{1, 6, 7\}$
Due Date: Friday October 10th, 2014.

1. Consider the matrix A to be one of the following:-

$$\begin{bmatrix} 2 & -1 & 1 & 3 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 2 & -2 \\ -1 & 1 & 0 & -2 \end{bmatrix}, \begin{bmatrix} 7 & -6 & 6 \\ 2 & 0 & 4 \\ 1 & -2 & 6 \end{bmatrix}.$$

For each choice of A:

(a) Find non-singular matrix $P_{n\times n}$ of order n and a diagonal non-singular matrix $D_{r\times r}$ such that

$$A = P \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} P^T$$

(b) There exists non-zero real numbers $d_1, \dots d_r$ (not necessarily disttinct) and orthonormal vectors $u_1, u_2, \dots u_r$ and $v_1, v_2, \dots v_r$ such that

$$A = \sum_{i=1}^{r} d_i u_i v_i^T,$$

$v_i^T u_j = 0$ if $i \neq j$ and 1 otherwise.

2. Let $A_{n\times n}$ be a Hermitian matrix. Show that its eigen values are real.

3. Let $A_{n\times n}$ be a real matrix with real eigen values. Show that it is orthogonally similar to an upper triangular matrix.

4. Let $A_{n\times n}$ be a real matrix. Show that it is orthogonally similar to diagonal matrix if and only if it is symmetric.

5. Let $A_{n\times n}$ be a real matrix with rank r. Show that

(a) There exists an orthogonal matrix $P_{n\times n}$ of order n and a real diagonal non-singular matrix $D_{r\times r}$ such that

$$A = P \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} P^T$$

(b) There exists non-zero real numbers $d_1, \dots d_r$ (not necessarily disttinct) and orthonormal vectors $u_1, u_2, \dots u_r \in \mathbb{R}^n$ such that

$$A = \sum_{i=1}^{r} d_i u_i u_i^T$$

6. Find a unitary matrix Q such that $Q^\star AQ$ is upper triangular and an orthogonal matrix P such that $P^T BP$ is diagonal when :-

$$A = \begin{bmatrix} -7 & -13 & -5 \\ 2 & 5 & -5 \\ -8 & -2 & 11 \end{bmatrix}, B = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & -2 & 8 \end{bmatrix}.$$

7. Compute a singular value decomposition of

$$A = \begin{bmatrix} 2 & 1 & 0 & 1 \\ -1 & 1 & 1 & -2 \\ 1 & 2 & 1 & -1 \end{bmatrix},$$