Problems due:X where $X \in$ *uniform*$\{2, 5\}$
Due Date: Wed October 1st, 2014.

1. Find the best-fit (least-squares[1]) line $y = \beta x + c$ for the points below.

x	1	2	4	5
y	1	2	2	3

2. (GS Ex 3.37, page 200) Find an othonormal basis for the column space of A and using the QR decomposition of A find the least square solution to $Ax = b$ where

$$A = \begin{bmatrix} 1 & -6 \\ 3 & 6 \\ 4 & 8 \\ 5 & 0 \\ 7 & 8 \end{bmatrix} \text{ and } b = \begin{bmatrix} -3 \\ 7 \\ 1 \\ 0 \\ 4 \end{bmatrix},$$

3. (BR Page 265)Consider the vector space $\mathcal{P}_4$ of all polynomials over $\mathbb{R}$ with degree atmost 3. Fix the set $\{a_1 = \frac{-3}{2}, a_2 = \frac{-1}{2}, a_3 = \frac{1}{2}, a_4 = \frac{3}{2}\}$ and for any $x, y \in \mathcal{P}_4$ define the inner product to be

$$\langle x, y \rangle = \sum i = 1^4 x(a_i) y(a_i)$$

   (a) Find an orthonormal basis for $\mathcal{P}_4$ by applying Gram-schmidt to $\{1, t, t^2, t^3\}$.
   (b) Find the polynomial of degree 3 that passes through the points $(\frac{-3}{2}, 1), (\frac{-1}{2}, 2), (\frac{1}{2}, 3)(\frac{3}{2}, 4)$.
   (c) Find the polynomial of degree 3 that passes through the points $(5, 15), (7, 18), (9, 25)(11, 26)$.
   (d) Describe the procedure to find best-fit polynomial of degree 2 that passes through the points $(\frac{-3}{2}, 1), (\frac{-1}{2}, 2), (\frac{1}{2}, 3)(\frac{3}{2}, 4)$.

4. Let $A_{m\times n}$ be a matrix with rank$(A) = r$. Find the rank$(AA^\star)$

5. (BR Ex 9 page 293)Find a rank-factorisation of the matrix

$$C = \begin{bmatrix} 2 & 4 & 2 & 4 & 4 \\ 1 & 2 & 1 & 2 & 2 \\ 3 & 0 & 3 & 3 & 0 \\ 0 & -4 & 0 & -2 & -4 \\ 5 & 2 & 5 & 6 & 2 \end{bmatrix}$$

   and hence its characteristics roots.

6. Determine Eigen-values and Eigen-spaces for

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix},$$

[1] I.E. the line or coefficients m, c that minimises the $\sum_{i=1}^4 (y_i - mx_i - c)^2$.