Problems due: 6
Due Date: Wednesday August 27th, 2014.

1. Prize question: Prove convexity of exponential function without using derivatives from calculus.

2. (BR Ex 2 page 190) Obtain a system $Ax = b$ for which the solution set is given by

$$S = \left\{ \begin{bmatrix} 1 + 4\alpha + 3\beta \\ 2 + 3\alpha \\ 1 + 8\beta \\ \alpha + 5\beta \end{bmatrix} : \alpha, \beta \in \mathbb{R} \right\}$$

3. (BR Ex 13 (c) page 192) Let $A_{m \times n}$ be a matrix with column space $\mathcal{C}(A)$. Let $b \in \mathcal{C}(A)$. Show that $Ax = b$ has a solution belonging to $\mathcal{C}(A)$ if and only if $m = n$ and $\text{rank}(A) = \text{rank}(A^2)$

4. (BR Ex 17 page 193) Suppose $A_{m \times n}x = b$ is consistent and $v^T \in \mathcal{R}(A)$, the row space of A. Then show that $(A + uv^T)x = b$ is consistent.

5. If $A_{m \times n}$ is a matrix with $m > n$ then does $Ax = 0$ always have a non-zero solution ?

6. Solve the system $Ax = b$, find the rank, rank-factorisation of A and a basis for the nullspace of A when

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 1 & 4 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$