Due: Wednesday, April 15th 2009

1. Determine the value and optimal strategies for the 2-person zero-sum game with payoff matrix

$$\begin{pmatrix} 2 & 3 \\ 5 & a \end{pmatrix},$$

   where $a \in \mathbb{R}$.

2. Consider the following LP. which is from a 2-person

$$\begin{array}{ll} \max & v \\ \text{subject to} & \sum_{j=1}^{n} a_{ij} p_j \geq v \text{ for } i = 1, \dots, m \\ & \sum_{i=1}^{m} p_i = 1 \\ & p \geq 0, v \in \mathbb{R} \end{array}$$

   Find the dual problem and use Lagrange's theorem to find sufficient conditions for existence of optimal strategy.

3. Let the matrix $A_{m\times n}$ define a two-person zero sum game. Show that there exist numbers $p_1, \dots p_m$ and $q_1, \dots, q_n$ with $p_i, q_j \geq 0, \sum_{j=1}^{n} p_j = \sum_{i=1}^{m} q_i = 1$ such that

$$\sum_{i,j} a_{ij} p'_i q_j \leq \sum_{i,j} a_{ij} p_i q_j \leq \sum_{i,j} a_{ij} p_i q'_j$$

   for all number $p'_1, \dots p'_m$ and $q'_1, \dots, q'_n$ with $p'_i, q'_j \geq 0, \sum_{j=1}^{n} p'_j = \sum_{i=1}^{m} q'_i = 1$ and explain why this implies the existence of optimal strategy for the game.

4. Each player selects a number from $1, 2, 3, 4, 5$. The players reveal their number and the player with the smaller number wins 2 rupees, unless the numbers are adjacent. In that case the player with the larger number wins 1 rupee. If the numbers are tied then the payoff is 0. Construct the payoff matrix for this zero sum game. Find the optimal strategy for player 2 .