Due Date: January 19th, 2008

Problems to be turned in: 3

1. Let $A, K \subset \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}$.
   (a) Suppose $n = 1$ and a is the least upper bound of A then show that there is a sequence a_k in A such that $a_k \to a$.
   (b) Suppose K is compact then show that for any sequence $\{x_n\} \in K$ there is a $x \in K$ and a subsequence $\{x_{n_k}\}$ such that $x_{n_k} \to x$.
   (c) Suppose f is continuous and $x_n \to x$ then show that $f(x_{n_k}) \to f(x)$.

2. Let $p_{n\times 1}$ be an element of $\mathbb{R}^n_+$ and $I > 0$. Let $B(p, I) = \{x \in \mathbb{R}^n_+ : p^T x \leq I\}$. Decide whether $B(p, I)$ is compact or not.

3. Let $c_{n\times 1} \in \mathbb{R}^n_+, y > 0$. Consider the cost minimisation problem ,

$$\begin{aligned} \text{Minimise} \quad & c_1x_1 + c_2x_2 \\ \text{Subject to} \quad & g(x) \geq y \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

   where the production function $g : \mathbb{R}^2_+ \to \mathbb{R}_+$ given by

$$g(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = x_1^2 + x_2^2.$$

   Decide whether the function can be solved by Lagrangian method or not.

4. Let $f : D \to \mathbb{R}$ be function on an open set D. Show that f is a C^1 function if and only if all partial derivatives exists on D and are continuous. Moreover in such a case,

$$Df(x) = \left[\frac{\partial f}{\partial x_j}(x)\right]$$

5. Let $f : \mathbb{R}^n \to \mathbb{R}^m$, $h : \mathbb{R}^k \to \mathbb{R}^n$. If h is differentiable at $x \in \mathbb{R}^n$ and f is differentiable at $h(x)$. Then $f \circ h : \mathbb{R}^k \to \mathbb{R}^m$ is differentiable at x and

$$D(f \circ h)(x) = Df(h(x))Dh(x).$$