Quiz 16th, February, 2006.

1. A person consumes three commodities. Suppose the utility function was given by:
$$u(x_1, x_2, x_3) = x_1^{\frac{1}{3}} + \min(x_2, x_3).$$
Given an income I, and prices of p_1, p_2, p_3. Is there a solution to the utilitiy maximisation problem ? If yes then can you use Kuhn-Tucker to characterise the solutions ?

2. A firm produces a single output y using three inputs x_1, x_2, x_3 in non-negative quantities through the relationship
$$y = g(x_1, x_2, x_3) = x_1(x_2 + x_3).$$
The unit price of y is $p_y > 0$, while that of the input x_i is $w_i > 0$, $i = 1, 2, 3$.
   (a) Describe the firm's profit maximisation problem, and derive the equations that define the critical points of the Lagrangean L in this problem.
   (b) Show that the Lagrangean L has multiple critical points for any choice of $(p_y, w_1, w_2, w_3, w_4)$.
   (c) Show that none of these critical points identifies a solution of the profit-maximisation problem. Explain why.

3. Let $\Phi : \Theta \to P(S)$ be a compact-valued, usc correspondence. Then, if $K \subset \Theta$ is compact, so is $\Phi(K) = \{t \in S : s \in \Phi(\theta) \text{ for some } \theta \in K\}$.

4. Let $\Phi : \Theta \to P(S)$ be a compact-valued correspondence. Then, Φ is usc at $\theta \in \Theta$ if and only if for all sequences $\theta_p \to \theta \in \Theta$ and for all sequences $s_p \in \Phi(\theta_p)$, there is a subsequence $s_{k(p)}$ of s_p such that $s_{k(p)}$ converges to some $s \in \Phi(\theta)$.

5. Let $D \subset R^n$ be compact and $f : D \to R$.
   (a) If f is usc on D (i.e. if for all sequences $x_k \to x \limsup_{x\to\infty} f(x_k) \leq f(x)$), it attains its supremum on D.
   (b) If f is lsc on D (i.e. if for all sequences $x_k \to x \liminf_{x\to\infty} f(x_k) \geq f(x)$), it attains is infimum on D.

6. Assume $\Phi : R \to P(R)$ be a correspondence. Determine in each of the following whether Φ is usc and/or lsc on R.
   (a) $\Phi(x) = [0, \frac{1}{x}]$ if $x > 0$ and $\Phi(x) = \{0\}$ if $x < 0$.
   (b) $\Phi(x) = \{\frac{1}{x}\}$ if $x > 0$ and $\Phi(x) = \{0\}$ if $x < 0$.
   (c) $\Phi(x) = [0, 1]$ if $x \neq 0$ and $\Phi(x) = (0, 1)$ if $x = 0$.

7. Let $S = [0, 2]$ and $\Theta = [0, 1]$. Let $f : S \times \Theta \to R$ be defined
$$f(x,\theta) = \begin{cases} 0 & \text{if } \theta = 0 \\ \frac{x}{\theta} & \text{if } \theta > 0, \text{ and } x \in [0, \theta) \\ 2 - (\frac{x}{\theta}) & \text{if } \theta > 0, \text{ and } x \in [\theta, 2\theta] \\ 0 & \text{if} x > 2\theta \end{cases}$$
Let the correspondence $D : \Theta \to P(S)$ be defined by
$$D(\theta) = \begin{cases} [0, 1 - 2\theta] & \text{if } \theta \in [0, \frac{1}{2}) \\ [0, 2 - 2\theta] & \text{if } \theta \in [\frac{1}{2}, 1] \end{cases}$$
Do f and D meet all the conditions of the Maximum Theorem ? Justify. Is it the case that $D(\theta) \neq \emptyset$? If yes then determine if it is usc or lsc on Θ