Quiz[1]:

1. Using Lagrangian multipliers, find the maxima and minima of the following fucntions subject to the specified constraints:
   (a) $f(x, y) = xy$ subject to $x^2 + y^2 = 2a^2$.
   (b) $f(x, y, z) = x^2 + 2y - z^2$ subject to $2x - y = 0, x + z = 6$.
   (c) $f(x, y) = x + y$ subject to $(x^2 - y^2)^2 = x^2 + y^2$.

2. Consider the following Pareto-optimal dicision of a given amount of resources between two agents. Given a weight $\alpha \in (0, 1)$, the problem is:

$$\text{Maximise} \quad \alpha x_1 y_1 + (1-\alpha) x_2 y_2 \tag{1}$$
$$\text{Subject to} \quad x_1 + x_2 \leq x \tag{2}$$
$$y_1 + y_2 \leq y \tag{3}$$
$$x_1 \geq 0, y_1 \geq 0, x_2 \geq 0, y_2 \geq 0 \tag{4}$$

   Solve this problem using Lagrange Multipliers, giving adequate justification.

3. Consider the cost minimisation problem with the production function g given by $g(x_1, x_2) = x_1^2 + x_2^2$. Decide whether the function can be solved by Lagrangian method or not.

4. Consider the utility maximisation problem with the utility function u given by $u(x_1, x_2) = x_1^\alpha + x_2^\alpha$, where $\alpha, \beta > 0$. Decide whether the problem can be solved by the Lagragian method.

5. Consider the Producer theory example. Solve

$$\begin{aligned} \text{Minimise} \quad & w_1 x_1 + w_2 x_2 \\ \text{Subject to} \quad & x_1 \geq 0 \\ & x_2 \geq 0 \\ & x_1^2 + x_2^2 - y \geq 0. \end{aligned}$$

[1] You need not turn in any of the problems. There will be a quiz on the due date mentioned which will feature a problem closely related to the above mentioned problems