Quiz[1]: January 19th, 2005

1. Let $D = [0, 1]$ Let f be an increasing function and g be a decreasing function on D. Convince yourself that f and g both attain their maximum and minimum on D. Does $f + g$ necessarily attain a maximum and minimum on D ?

2. (**Profit maximisation**) Suppose Suppandi-Kengeri corporation produces $y = g(x_1, x_2, \ldots, x_n)$ units of output using x_i units of the $i - th$ input. Let $c = (c_1, c_2, \ldots, c_n)$ be the cost vector. When the firm produces y units, the unit price it can obtain for the product is given by $p(y)$. You are assigned the job of choosing an input mix that will maximise the Suppandi-Kengeri profits. I.e. find a $x \in \mathbb{R}^n_+$ that solves:

$$\max\{p(g(x))g(x) - c^T x | x \in \mathbb{R}^n_+$$

   Assume that

   (a) $g : \mathbb{R}^n_+ \to \mathbb{R}_+$, the firm's production technology is continuous.
   (b) The unit price c_i for input i is strictly positive for all i.
   (c) The market inverse demand curve $p : \mathbb{R}_+ \to \mathbb{R}_+$ is a constant function, whose value is $p > 0$.

   Are the above assumptions enough for you to prove that there exists a solution to the profit maximisation problem of the firm ?

3. A fishery earns a profit of $\pi(x)$ from catching and selling x units of fish. The firm owns a pool which currently had y_1 fish in it. If $x \in [0, y_1]$ fish are caught this period, the remaining $i = y_1 - x$ fish will grow to $f(i)$ fish by the begining of the next period, where $f : \mathbb{R}_+ \to \mathbb{R}_+$ is the growth function for the fish population. The fishery wishes to set the volume of its catch in each of the next three periods so as to maximize the sum of its profits over this horizon. That is, it solves:

$$\begin{array}{rc} \text{Maximise} & \pi(x_1) + \pi(x_2) + \pi(x_3) \\ \text{subject to} & x_1 \leq y_1 \\ & x_2 \leq y_2 = f(y_1 - x_1) \\ & x_3 \leq y_3 = f(y_2 - x_2) \\ & (x_1, x_2, x_3) \in \mathbb{R}^n_+ \end{array}$$

   Show that if π and f are continuous on $\mathbb{R}_+$, then there is a solution exists to this problem.

[1]You need not turn in any of the problems. There will be a quiz on the due date mentioned which will feature a problem closely related to the above mentioned problems