1. Manually perform three steps of Euler's method to solve

$$\frac{dy}{dt} = \frac{1}{t+y+1}, \ y(0) = 0$$

with h = 0.2.

2. Consider

$$\frac{dy}{dt} = t - 2y, \ y(0) = 1.$$

Compute its exact solution. Using rhs1 and odeEuler compute the numerical solution using Euler's method in [0,0.6] with h=0.2. Use demoEuler to compare the numerical solution with the exact solution. Further compute the numerical solution using Euler's method in [0,0.6] with h = 0.1 and h = 0.05. Plot all three solutions along with the exact solution.

3. Consider

$$\frac{dy}{dt} = y, \ y(0) = 10 \le t \le 1$$

Compute its exact solution. Using rhs2, odeMidpt compute the numerical solution using the midpoint method in [0,1] with h=0.2. Use compEM to compare this numerical solution with that obtained by Euler's method. For the same accuracy comment on the number of flops required between Euler versus Midpoint method.

4. Consider

$$\frac{dy}{dt} = y, \ y(0) = 10 \le t \le 1$$

Compute its exact solution. Using rhs2, odeRK4 compute the numerical solution using the Runge-Kutta method in [0,1] with h=0.2. Use compEMRK4 to compare this numerical solutions obtained by Euler, Midpoint and Runge-Kutta (4) method. For the same accuracy comment on the number of flops required between these methods.

5. Use (odeEuler or otherwise) Euler's method to with h = 0.05 to solve

$$\frac{dy}{dt} = \sqrt{y}, \ y(0) = 0, \ 0 \le t \le 2$$

Recompute the solution using odeMidpt and odeRK4. Plot a comparison of the numerical solution(s) with the exact solution. Does the plot indicate an error in odeEuler?

6. Starting with odeEuler, suitably modify it to write an function that will implement Heun's mehthod for an arbitrary first-order ODE. Use your function to solve

$$\frac{dy}{dt} = t - 2y, \ y(0) = 1$$

for h = 0.2, 0.1, 0.05 and h = 0.025.