Due: November 8th, 2005.

1. Let X be an integrable random variable on a probability space,$(\Omega, \mathcal{B}, P)$ and $\mathcal{C}$ be a sub-σ algebra of $\mathcal{B}$. Then the conditional expectation of X given $\mathcal{C}$, denoted by $E(X \mid \mathcal{C})$, is a $\mathcal{C}$ measurable random variable Y such that
$$\int_C Y dP = \int_C X dP \ \forall C \in \mathcal{C}.$$
   (a) Show that $E(X \mid \mathcal{C})$ exists. (*Hint:* Use Radon-Nikodym Theorem).
   (b) Suppose $\sigma(X)$ is independent of $\mathcal{C}$ then
   $$E(X \mid \mathcal{C}) = E(X).$$
   (c) Let Y be a random variable on $(\Omega, \mathcal{C})$,such that XY is integrable then
   $$E(XY \mid \mathcal{C}) = YE(X \mid \mathcal{C}).$$
   (d) If $\mathcal{D}$ is another sub-σ algebra, $\mathcal{D} \subset \mathcal{C} \subset \mathcal{G}$ then
   $$E(E(X \mid \mathcal{C}) \mid \mathcal{D}) = E(X \mid \mathcal{D}) = E(E(X \mid \mathcal{D}) \mid \mathcal{C})$$
2. Problem 2 Page 57 Rudin, Real and Complex Analysis.
3. Problem 17 Page 59 Rudin, Real and Complex Analysis.
4. Problem 20 Page 59 Rudin, Real and Complex Analysis.