Due: Thursday October 25th, 2005
Problem 1,2,3

1. Let $(\Omega, \mathcal{B}, P)$ be a probability space. Let $\phi : \mathbb{R} \to \mathbb{R}$ be a convex function. If $X : \Omega \to \mathbb{R}$ is integrable then show that
$$\phi(E(X)) \leq E(\phi(X)).$$
2. Let $(\Omega, \mathcal{B}, \mu)$ be a measure space. Show that $L^\infty(\Omega, \mathcal{B}, \mu)$ is complete metric space with the metric coming from the essential supremum norm.
3. Let $1 \leq p \leq \infty$ and if $\{f_n\}$ is a cauchy sequence in $L^p(\mu)$ with limit f then f_n has a subsequence which converges pointwise almost everywhere to f.
4. Page 74, Problem 19, in Real and Complex Analysis, Rudin
5. Page 74, Problem 23, in Real and Complex Analysis, Rudin